R-estimation in linear models Algorithms, complexity, challenges

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The main objective of this lecture is to discuss selected computational aspects of robust estimation in the linear model with the emphasis on R-estimators. We focus on numerical algorithms and computational efficiency rather than on statistical properties. In addition, we formulate some algorithmic properties that a "good" method for R-estimators is expected to meet and show how to satisfy them using the currently available algorithms. We illustrate both good and bad properties of the existing algorithms. We propose two-stage methods to minimise the effect of the bad properties. Finally, we justify a challenge for new approaches based on interior-point methods in optimisation.

We will concentrate on the description of selected algorithms, and from our point of view, the most interesting ones, intended to find robust R-estimators in a linear regression model. These algorithms can be divided into two general classes, namely, exact (those who can determine whether the current solution is a true minimiser) and approximate ones. Although exact algorithms should be preferred, the emphasis is on approximate algorithms, which are both prevailing and, from the practical point of view, more easily interpretable.

It appears that the future should be in a combination of these two approaches, leading to the creation of two-step procedures. While in the first step a properly selected approximate algorithm would result in the suboptimal solution lying close to the true minimiser, in the second step an exact algorithm should find the true minimiser. Furthermore, we can see a challenge in developing a special interior point method for Osborne's LP formulation, which could have the potential to overcome most of the difficulties associated with current algorithms.

Key words: Linear regression model; R-estimators; Least squares estimator; L1-norm estimator; Iteratively reweighted least squares; Iterated weighted least squares; s-step R-estimators; Newton-like algorithm; Line and simplex search; Discrete and continuous optimization; Arrangement of hyperplanes

References

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