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When is a CAT index futures traded and preferred to reinsurance? — Trade-off between basis risk and adverse selection —

Kazuhiko Ohashi

The Graduate School of International Corporate Strategy,
Hitotsubashi University

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When is a CAT index futures traded and preferred to reinsurance?
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Abstract

Insurance risks are traditionally borne in reinsurance markets. In 1990s, however, after a sequence of huge natural disasters and huge insurance payments to them, the reinsurance markets reduced their capability to bear risks. Insurance-linked securities were created to provide the insurers the way to transfer insurance risks to the capital markets. A CAT (catastrophe) index futures is one of them. There are, however, obstacles that prevent the CAT index futures from being traded: basis risk between the insurers' risks and the payoff of the futures and adverse selection between the informed insurers and the uninformed investors. In this situation, this paper investigates conditions under which the CAT index futures, whose payoff is the average of insurers' losses, can be traded and preferred to the reinsurance. It shows that the index futures is traded if the number of insurers in the index is large enough since averaging many enough insurers' losses mitigates adverse selection in the payoff of the index futures. It on the other hand shows that if the number of insurers in the index is too large, the insurers prefer the reinsurance to the index futures due to large basis risk in the futures' payoff.

JEL classification: G14, G22, G23

Keywords: insurance-linked securities (ILS), reinsurance, CAT (catastrophe) futures and options, basis risk, adverse selection.

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1. INTRODUCTION

Insurance risks are traditionally borne in reinsurance markets. That is, after originating insurance contracts and bearing risks of customers, the insurers spread and reallocate such insurance risks among themselves and reinsurers.

In 1990’s, however, the capability of the reinsurance markets to bear risks was at stake. A sequence of huge natural disasters, including Hurricane Andrew and Northridge Earthquake, produced huge amount of industry-wide insured losses, more than $30 billion just for Hurricane Andrew and Northridge Earthquake. There was the need for additional reinsurance capacity. Capital markets, or the non-insurance investors, became the natural target. (Ganapati et. al. (1997))

Then, innovation occurred. Insurance-linked securities (ILS's) were created, which enabled the investors outside the insurance and reinsurance industry to trade the catastrophe insurance risks so that the capital markets could provide additional capacity for bearing the insurance risks. CAT (catastrophe) bonds traded in over the counter (OTC) markets and/or CAT index futures and options traded in organized exchanges are among those ILS's. (Cummins and Barrieu (2013))

Since the mid 1990’s, CAT bond markets have grown steadily. The amount of issuance achieved almost $7.2 billion in 2007 and, though it reduced to $3 billion in 2008 due to the financial crisis, exceeded $7.9 billion in 2014. (Guy Carpenter (2015)) Despite the success of CAT bonds, however, interests in CAT index futures and options have been quite limited so far. The CAT futures and options introduced by the Chicago Board of Trade (CBOT) and the Bermuda Commodity Exchange (BCOE) in 1990’s failed soon after their launch. In 2000’s, the Insurance Futures Exchange (IFEX), the New York Mercantile Exchange (NYMEX), and the Chicago Mercantile Exchange (CME) reintroduced the CAT futures and options, but none is traded in 2016.

What is wrong? Behind such apparent failure of CAT futures and options lie two unavoidable obstacles for securitization of insurance risks; basis risk and adverse selection. Basis risk is the risk that the payoff of a security is not perfectly correlated with the hedger’s loss. Hence, by trading an ILS with basis risk, an insurance company has to assume additional risk that is unrelated to its hedging need. On the other hand, adverse selection is the risk that the seller of a security has advantageous private information about the value of the security than the buyer. If the adverse selection problem is too serious, the buyer refrains from trading the security with the seller. Note that all ILS’s including CAT bonds and CAT futures/options are subject to these problems. Whether their markets take off successfully depends on how well these problems can be mitigated by designing appropriate structure of the securities.[2]

In designing an ILS, its payoff is defined as a function of variables that are related to losses of insurers. Accordingly, the payoff of a CAT index futures/options is determined by a designated index. For example, the CAT futures introduced by CBOT and IFEX used the indexes provided by Property Claims Services (PCS), and the one by CME used the Carvill Hurricane Index. These indexes, however, are designed to reflect either industry-wide insurance losses or a specified physical severity of the event, which is supposed to be related to insurers' losses, but do not exactly represent the individual insurers' losses. From the purpose of risk-reallocation, such design of the indexes limits the hedging effectiveness of the CAT index derivatives for the individual insurers. The risk traded by such index is inevitably different from the risks that the individual insurers wish to hedge. This is basis risk.

Why do the exchanges create derivatives with such basis risk? If the insurers and the investors outside the insurance industry have the same information about the insurance risks, there should be no benefit for creating index derivatives for both sides. Trading each insurer's risk individually avoids the basis risk in trading the index and provides better risk-sharing opportunities. However, this is not the case for insurance risks. Usually, the outside non-insurance investors know little about the risks that the insurance contracts deal with, while the insurers and the reinsurers know them well. Adverse selection problem is prevalent between the insiders and the outsiders of the insurance markets. Trading an index of insurance risks is expected to mitigate such adverse selection and to facilitate the participation of the non-insurance investors in trading. (Kist and Meyers (1999), Major (1999), and Cummins and Barrieu (2013).)

Despite such structural device, the CAT index derivative markets have never been successful. For all contracts, the markets have never taken off. Although this may be partly because of the recovery of capacity of the reinsurance markets, from the theoretical viewpoint, the developments of the CAT index futures/options should be meaningful steps in achieving incremental risk transfer to the capital markets.

Thus, this paper investigates the condition under which the index derivatives, such as the CAT index futures, of the insurance risks can be traded and preferred to the reinsurance contracts. In what follows, we focus on the situation where each insurance

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[2] Cummins and Barrieu (2013) point our moral hazard, instead of adverse selection, as one of the obstacles for securitizing insurance risks. In fact, both adverse selection and moral hazard are important problems in trading/contracting under asymmetric information. In this paper, we focus on adverse selection about insurance risks as a primary problem between insurers and outside investors.

risk of each insurer cannot be traded individually with the non-insurance outside investors due to serious adverse selection. (Each insurer knows too precisely about its own insurance risk than the outside investors.) As a way to mitigate such adverse selection problem, we consider an index futures where the index is defined to be the average of the payoffs of insurance risks that the insurers are endowed with.

We formulate a model that describes this situation and first investigate the condition under which averaging the insurers' losses to determine the payoff of an index futures mitigates the adverse selection problem enough so that the index futures can be traded between the informed insurers and the uninformed non-insurance investors. We show that the index futures can be traded if the insurers' trading is more motivated by hedging than information and that this condition is satisfied when the number of insurers, whose losses are included in the index, is large enough. This is because averaging the insurers' risks reduces the adverse selection in the futures' payoff by making the payoff less sensitive to the insurers' private information through diversification.

We then compare the attractiveness of the index futures to the reinsurance from the insurers' viewpoints. Participants in the reinsurance markets are presumably professionals in insurance. They know equally well about the insurance risks that they trade. Adverse selection problem about the insurance risks is small in the reinsurance markets. Thus, we model the reinsurance markets as one representative reinsurer who has the same information as the insurers, but is less risk tolerant than the representative non-insurance investor. (This reflects the fact that the reinsurer's capital is more limited than the investors'.) We consider the idealized reinsurance markets where the insurers and the reinsurer have the symmetric information, trade their risks competitively, and attain the optimal risk allocation. We then investigate the condition under which each insurer ex-ante wishes to trade the index futures rather than the reinsurance contracts. We find that if the number of insurers in the index is too large, the insurers prefer the reinsurance to the index futures. This is because the industry-average of the insurers' losses is inevitably different from the loss of the individual insurer and hence averaging the losses over too many insurers exacerbates the basis risk in the payoff of the index futures.

This paper is related to numerous papers on securitization of catastrophe risk, especially on the effect of basis risk and moral hazard/adverse selection and on the use of CAT instruments and reinsurance. For example, Doherty (1997) argues that managing the tradeoff between basis risk and moral hazard is the key to the success in securitization of catastrophe risk. Doherty and Richter (2002) show that the insurer should use a combination of CAT index instrument and reinsurance to reduce basis risk. Harrington and Niehaus (1999) empirically find that the basis risk of state-specific CAT index derivatives are effective hedging tools in that their basis risk is not too large for the insurers. Cummins, Lalonde, and Phillips (2004) analyze the effectiveness of CAT index options in hedging hurricane losses for the Florida insurers and find that hedging with a statewide loss index is effective for the large and medium insurers, but not for the small insurers. Barrieu and Louberge (2009) show that a CAT bond can be more popular if it is combined with an option that protects bond buyers against simultaneous drops in stock prices with catastrophe occurrence. Nell and Richter (2004) predict that the CAT instruments substitute reinsurance for large losses due to risk aversion of reinsurers. Subramanian and Wang (2014) show in a signaling model that the lowest risk insurers choose reinsurance while intermediate and high risk insurers choose partial and full securitization and hence that catastrophe risks are securitized by high risk insurers. Gibson, Habib, and Ziegler (2014) investigate an insurer's choice between issuing a CAT bond and using reinsurance when the insurer can subtract information from the investors trading the bond and the reinsurers. They show that when there are many liquidity traders in the bond market or when the insurer's loss is highly uncertain, the cost of issuing a CAT bond becomes too large and reinsurance is preferred by the insurer.

This paper complements the extant literature by analyzing the tradeoff between basis risk and adverse selection in the trade of a CAT index futures between the insurers and the investors and in the insurers' choice between a CAT index futures and reinsurance. Here, basis risk is the difference between the risks that the insurers are exposed to and the payoff of the index futures. Adverse selection is that the insurers are more informed than the investors. Determining the payoff of the CAT index futures as the average of insurers' losses is a device to mitigate the adverse selection because by averaging, the payoff of the futures becomes less sensitive to the individual insurers' private information. This enables the CAT index futures to be traded even under strong adverse selection on the insurers' individual risks, but also exacerbates the basis risk so that reinsurance is preferred to the index futures if averaging is done over too many insurers.

This paper also shares ideas and modelling devices with the papers on financial innovation. For example, the mechanism that averaging payoffs that are subject to private information reduces adverse selection is closely related to that of Subrahmanyam (1991). Strong adverse selection under which each informed insurer's endowment risk cannot be traded with the uninformed investor is described by utilizing the models of Bhattacharya, Reny, and Spiegel (1995), Bhattacharya and Spiegel (1991), Rahi (1996) and Ohashi (1999). Marin and Rahi (1999) are instructive to obtain ex-ante utility level of a negative exponential (CARA) utility function with normally distributed shocks.[3]

The organization of this paper is as follows: In section 2, we formulate the model. In section 3, we show the condition under which each insurer's endowment risk cannot be traded with the outside investors individually due to strong adverse selection. In section 4, we investigate the condition under which the index futures can be traded between the insurers and the investors despite the strong adverse selection. In section 5, we analyze the risk sharing through the reinsurance markets. In section 6, we obtain each insurer's utility when he trades the index futures and when he shares risks through the reinsurance markets. In section 7, we provide several numerical examples. In section 8, we conclude with some remarks on further investigation.

2. THE MODEL

All random variables are defined on a probability space $(\Omega, \mathcal{F}, P)$. Throughout we denote by $\mathrm{Var}[\chi]$ the unconditional variance of random variable χ, and by $\mathrm{Var}[\chi|\eta]$ the conditional variance of χ given η. Similarly, $\mathrm{Cov}[\chi, \xi]$ denotes the unconditional covariance between of χ and ξ, and $\mathrm{Cov}[\chi, \xi|\eta]$ denotes their conditional covariance given η..

There are $N + 2$ agents, with von Neumann-Morgenstern utility functions displaying constant absolute risk aversion. We refer to the first N agents insurers and assume that each insurer n $(n = 1, \dots, N)$ has an asset, as an insurance contract, which yields a stochastic payoff z_n at the terminal date. The $N + 1$st agent is an aggregated reinsurer who provides reinsurance for the insurers. The $N + 2$nd agent is an aggregated outside investor who represents the non-insurance investors in the capital markets as a whole.

There are three dates in the economy. At the ex-ante stage, date 0, the insurers decide which contract and with whom they trade at date 1; an index futures with the outside investors or reinsurance contracts with the reinsurer. At the interim stage, date 1, the insurers and the reinsurer receive some private information signals about the payoffs of the insurance risks, while the outside investors receives no signal. Right after the information signal is received, the insurers trade either an index futures with the

[3] For more about the literature on financial innovation, see e.g., Allen and Gale (1994), DeMarzo and Duffie (1999), and Duffie and Rahi (1995).

uninformed outside investor, or reinsurance contracts with the reinsurer, as they planned at date 0. At the final stage, date 2, payoffs are realized and all signals become public.

More precisely, the insurer $n\ (n = 1, \dots, N)$ has the utility function $E[-\exp(-\frac{1}{\gamma_I}W)]$ over the consumption W at date 2, is endowed with an asset that has a payoff z_n at date 2, and receives a vector S of information signals at date 1. The reinsurer has the utility function $E[-\exp(-\frac{1}{\gamma_R}W)]$ over the consumption W at date 2, receives the same vector S of information signals that the insurers do at date 1, but has no endowment. The outside investor has the utility function $E[-\exp(-\frac{1}{\gamma_u}W)]$ over the consumption W at date 2, but has neither endowment of an asset nor an information signal.

For tractability, we assume that z_n is given by a product of two random variables x_n and e_n, namely $z_n = x_n e_n$, where x_n represents the per unit payoff of the n's insurance risk and e_n represents the size, in terms of units, of the n's insurance risk. We also assume that $x_n = x_c + \varepsilon_n$ where x_c is a common market-wide risk and ε_n is the insurer n's individual insurance risk.

For each n, denote by s_n a signal on ε_n. Define $e \equiv [e_1, \dots, e_N]^\intercal$, $\varepsilon \equiv [\varepsilon_1, \dots, \varepsilon_N]^\intercal$, $x \equiv [x_1, \dots, x_N]^\intercal$, and $s \equiv [s_1, \dots, s_N]^\intercal$. The vector of information signals S that the insurers and the reinsurer receives at date 1 is given by $S \equiv (s^\intercal, e^\intercal)^\intercal$.

All underlying random variables are normally distributed with 0 means, and are independent except that for each n, $\mathrm{Cov}[\varepsilon_n,\ s_n] = \rho\ (\rho \neq 0)$. We assume that $\mathrm{Var}[e_n]$ is the same for all n, that $\mathrm{Var}[\varepsilon_n]$ is the same for all n, and that $\mathrm{Var}[s_n]$ is the same for all n.

There are two kinds of possible contracts that the insurers choose to trade. One is an index futures contract with a payoff $F = \frac{1}{N}\Sigma_{n=1}^{N} x_n$. The other is reinsurance contracts with payoffs $x \equiv [x_1, \dots, x_N]^\intercal$. The insurers decide collectively whether they trade the index futures or the reinsurance contracts. In the former case, the insurers share their endowment risks with the uninformed non-insurance investor in the capital markets through trading the index futures. In the latter case, the insurers share their endowment risks with the equally informed reinsurer through the reinsurance markets.

Note that although the investor in the capital markets is less informed than the insurers and the reinsurer, typically the capital markets are much larger than the reinsurance markets in its size. Thus, with symmetric information, the aggregate investor of the larger capital markets would be more risk tolerant than the reinsurer in the smaller reinsurance markets, which suggests that $\gamma_u \geq \gamma_R$.

3. THE CONDITION FOR NO-TRADE OF INDIVIDUAL INSURANCE RISKS BY STRONG ADVERSE SELECTION

We will investigate the case where the insurers trade the index futures with the outside investor in the capital markets. We would like to clarify the condition under which bundling the insurers' endowment risks into an index futures facilitates risk sharing between the insurers and the outside investor.

Note that if the insurers and the outside investor have the same information about the risks that they trade, there is no benefit for creating an index futures for both sides. Trading each insurer's endowment risk separately provides better opportunities for risk sharing.

However, this is not the case for typical insurance markets, including CAT insurance. Usually, the outside non-insurance investor knows little about the risks that the insurance contracts deal with, while the insurers know them well. Asymmetric information is prevalent between the insiders and the outsider of the insurance markets.

Thus, we focus on the situation where each insurer's endowment risk cannot be traded individually with the outside investor because of strong adverse selection (or because the insurers know too precisely about their endowment risk), but bundling these risks into an index futures (or averaging the insurers' losses to determine the index futures' payoff) mitigates the adverse selection problem so that the index futures can be traded between the insurers and the outside investor.

For this purpose, let us start with the hypothetical case where each insurer n creates an insurance futures contract with the payoff x_n aiming to trade it with the outside investor. We assume that each insurer does not trade the other insurers' contracts.

Denote by θ_n the position that the insurer n takes in the insurance futures that it creates. Each insurer n tries to trade the futures x_n with the outside investor strategically i.e., taking account of the price impact of his position θ_n. Let $\theta = (\theta_1, \ldots, \theta_N)^\intercal$. We assume that the price is given by a linear function

$$P_n(\theta) = h_n + k\theta_n + l\sum_{m \neq n} \theta_m. \tag{1}$$

The insurer n's date 2 wealth after trading this futures is given by $W_n = e_n x_n + \theta_n\big(x_n - P_n(\theta)\big)$. At date 1, the insurer n solves the following problem to obtain the optimal position θ_n:

$$\mathrm{Max}_{\theta_n} E[-\exp(-\frac{1}{\gamma_I} W_n)|S].$$

Its first order condition is

$$E[x_n|S] - h_n - 2k\theta_n - l\sum_{m\neq n}\theta_m - \frac{1}{\gamma_I}\{\mathrm{Var}[x_n|S]e_n + \mathrm{Var}[x_n|S]\theta_n\} = 0. \quad (2)$$

and its second order condition is

$$2k + \frac{1}{\gamma_I}\mathrm{Var}[x_n|S] > 0. \quad (3)$$

Note that the optimal θ_n's are simultaneously determined. Define $q_n \equiv E[x_n|S] - \frac{1}{\gamma_I}\mathrm{Var}[x_n|S]e_n$, $Q \equiv (q_1, \dots, q_N)^\intercal$, and $H \equiv (h_1, \dots, h_N)^\intercal$. Denote by $I_{N\times N}$ the $N \times N$ identity matrix and by J_N the $N \times N$ matrix whose elements are all 1. Then, if the second order conditions are satisfied for all n, then the vector of the insurers' optimal positions $\theta = (\theta_1, \dots, \theta_N)^\intercal$ is given by

$$\theta = \frac{1}{2k+\frac{1}{\gamma_I}\mathrm{Var}[x_n|S]-l}\{I_N - \frac{1}{2k+\frac{1}{\gamma_I}\mathrm{Var}[x_n|S]+(N-1)l}J_N\}(Q-H). \quad (4)$$

Denote by θ_{un} the position that the outside investor u takes in trading the nth futures. Let $\theta_u = (\theta_{u1}, \dots, \theta_{uN})^\intercal$ and $x = (x_1, \dots, x_N)^\intercal$. The outside investor is assumed to represent a large number of identical investors and hence behaves competitively. He has rational expectations and uses the observed prices to update beliefs about the payoff of the traded securities. Then, the outside investor's date 2 wealth after trading the futures is given by $W_u = \theta_u^\intercal(x - P(\theta))$ where $P(\theta) = (P_1(\theta), \dots, P_N(\theta))^\intercal$. At date 1, the outside investor solves the following problem to obtain the optimal position θ_u:

$$\mathrm{Max}_{\theta_u} E[-\exp(-\frac{1}{\gamma_u} W_u)|P(\theta)].$$

Its first order condition is

$$E[x|P(\theta)] - P(\theta) - \frac{1}{\gamma_u}\mathrm{Var}[x|P(\theta)]\theta_u = 0. \quad (5)$$

and its second order condition is

$$\frac{1}{\gamma_u}\mathrm{Var}[x|P(\theta)] > 0. \quad (6)$$

This second order condition is satisfied by assumption. Hence, the vector of the outside investor's optimal position is given by

$$\theta_u = \gamma_u \mathrm{Var}[x|P(\theta)]^{-1}\{E[x|P(\theta)] - P(\theta)\}. \tag{7}$$

A linear rational expectation equilibrium is a set $(P(\cdot), \theta, \theta_u)$ such that (a) $P(\cdot)$ is given by (1), (b) agents maximize their utility, and (c) the markets clear, that is $\theta + \theta_u = 0$. From the insurers' second order condition (3), we obtain the following condition for the individual insurance risks to be traded separately in the capital markets:

Lemma 1:

An equilibrium exists if and only if

$$\Lambda \equiv \frac{1}{\gamma_I^2}\mathrm{Var}^2[x_n|s_n]\mathrm{Var}[e_n] - \frac{\mathrm{Cov}^2[\varepsilon_n, s_n]}{\mathrm{Var}[s_n]} > 0. \tag{8}$$

Lemma 1 shows that if $\Lambda \leq 0$, this economy fails to have an equilibrium. The first term of Λ is related to the hedging demand of the informed insurers as indicated by its dependence on the degree of risk tolerance. The second term is related to the informational motive for trading. Thus, an equilibrium exists if the hedging demand (the former) dominates the demand motivated by information (the latter). If this is not the case, the adverse selection problem is so severe that no equilibrium exists. Hence, throughout the rest of this paper, we assume the following:

Assumption 1: $\Lambda \leq 0$. (9)

That is, we focus on the situation where each insurer n cannot trade his own endowed insurance risk individually with the non-insurance outside investor because of the strong adverse selection between them.

4. THE INDEX FUTURES

One practical way to mitigate this adverse selection problem is to create an index of the average of the payoffs of the insurers' endowment risks and to create a futures contract whose payoff is determined by this index. (See Ganapati et. al. (1997), Kist and Meyers (1999), and Major (1999).) In this average index, it is expected that by the law of large number, each insurers' specific risks would be diversified away enough, if the number of insurers in the index is large enough. The index would, then, depend largely on the market-wide common risk. Since informational asymmetry is much less for the market-wide common risk than for the insurers' specific risks, the adverse selection for the average index futures is mitigated and much less than that for the individual insurance contracts.

Let $F = \frac{1}{N}\Sigma_{n=1}^{N} x_n$ be the payoff of the average index futures where we assume $x_n = x_c + \varepsilon_n$ where x_c is a common market-wide risk and ε_n is the insurer n's individual insurance risk. Denote by θ_{F_n} the position that the insurer n takes in trading the index futures. Let $\theta_F \equiv (\theta_{F_1}, \ldots, \theta_{F_N})^\intercal$. Each insurer n trades the index futures F strategically with the outside investor i.e., taking account of the price impact of his position θ_{F_n}. We assume that the price of the index futures is given by a linear function

$$P_F(\theta_F) = h_F + k_F \sum_{n=1}^{N} \theta_{F_n}. \tag{10}$$

The insurer n's date 2 wealth after trading the index futures is given by $W_{F_n} = e_n x_n + \theta_{F_n}\big(F - P_F(\theta_F)\big)$. At date 1, the insurer n solves the following problem to obtain the optimal position θ_{F_n}:

$$\mathrm{Max}_{\theta_{F_n}} E[-\exp(-\frac{1}{\gamma_I} W_{F_n})|S].$$

Its first order condition is

$$E[F|S] - h_F - 2k_F\{2\theta_{F_n} + \textstyle\sum_{m\neq n}\theta_m\} - \frac{1}{\gamma_I}\{\mathrm{Cov}[F, x_n|S]e_n + \mathrm{Var}[F|S]\theta_{F_n}\} = 0. \tag{11}$$

and its second order condition is

$$2k_F + \frac{1}{\gamma_I}\mathrm{Var}[F|S] > 0. \tag{12}$$

Define $q_{F_n} \equiv E[F|S] - \frac{1}{\gamma_I}\mathrm{Cov}[F, x_n|S]e_n$, $Q_F \equiv \left(q_{F_1}, \ldots, q_{F_N}\right)^\intercal$, and $H_F \equiv (h_F, \ldots, h_F)^\intercal$. Denote by $I_{N\times N}$ the $N \times N$ identity matrix and by J_N the $N \times N$ matrix whose elements are all 1. Then, if the second order conditions are satisfied for all n, then the vector of the insurers' optimal positions $\theta_F = \left(\theta_{F_1}, \ldots, \theta_{F_N}\right)^\intercal$ is given by

$$\Theta_F = \frac{1}{k_F + \frac{1}{\gamma_I}\mathrm{Var}[F|S]}\{I_N - \frac{k_F}{(N+1)k_F + \frac{1}{\gamma_I}\mathrm{Var}[F|S]} J_N\}(Q_F - H_F). \tag{13}$$

Define $\theta_{FI} \equiv \sum_{n=1}^{N} \theta_{F_n}$. Then, θ_{FI} is the aggregate position in the index futures by the insurers, and

$$\theta_{FI} = \frac{1}{(N+1)k_F + \frac{1}{\gamma_I}Var[F|s]} \left\{ NE[F|S] - \frac{1}{\gamma_u} Cov[F, x_n|S] \sum_{n=1}^{N} e_n - Nh_F \right\}. \tag{14}$$

Denote by θ_{Fu} the position that the outside investor u takes in trading the index futures. The outside investor behaves competitively. He has rational expectations and uses the observed price to update beliefs about the payoff of the index futures. Then, the outside investor's date 2 wealth after trading the index futures is given by $W_{F_u} = \theta_{F_u}^{T}\left(x - P_F(\theta_F)\right)$. At date 1, the outside investor solves the following problem to obtain the optimal position θ_{F_u}:

$$Max_{\theta_{F_u}} E[-\exp(-\frac{1}{\gamma_u} W_{F_u})|P_F(\theta_F)].$$

Its first order condition is

$$E[F|P_F(\theta_F)] - P_F(\theta_F) - \frac{1}{\gamma_u} Var[F|P_F(\theta_F)]\theta_{F_u} = 0. \tag{15}$$

and its second order condition is

$$\frac{1}{\gamma_u} Var[F|P_F(\theta_F)] > 0. \tag{16}$$

This second order condition is satisfied by assumption. Hence, the vector of the outside investor's optimal position is given by

$$\theta_{F_u} = \gamma_u Var[F|P_F(\theta_F)]^{-1}\{E[F|P_F(\theta_F)] - P_F(\theta_F)\}. \tag{17}$$

A linear rational expectation equilibrium is a set $(P_F(\cdot), \theta_F, \theta_{F_u})$ such that (a) $P_F(\cdot)$ is given by (10), (b) agents maximize their utility, and (c) the markets clear, that is $\theta_{FI} + \theta_{F_u} = 0$. From the insurers' second order condition (12), we obtain the following condition for the index futures to be traded:

Lemma 2:

An equilibrium exists for the index futures market if and only if

$$\Lambda_F \equiv \frac{1}{\gamma_I^2}\{(N-1)Var[x_c] + Var[x_n|s_n]\}^2 Var[e_n] - \frac{Cov^2[\varepsilon_n, s_n]}{Var[s_n]} > 0. \tag{18}$$

Similarly to lemma 1, the first term of Λ_F is related to the hedging demand of the informed insurers for the index futures and the second term is related to the informational motive for trading. An equilibrium exists if the former dominates the latter. Comparing this lemma with lemma 1, we now obtain the following result:

Proposition 1:

Between the insurers and the outside investor, the average index futures is traded, while the individual insurance risks cannot be traded separately, if and only if

$$\frac{1}{\gamma_I^2}\{(N-1)\mathrm{Var}[x_c]+\mathrm{Var}[x_n|s_n]\}^2\mathrm{Var}[e_n]>\frac{\mathrm{Cov}^2[\varepsilon_n,s_n]}{\mathrm{Var}[s_n]}>\frac{1}{\gamma_I^2}\mathrm{Var}^2[x_n|s_n]\mathrm{Var}[e_n]. \tag{19}$$

Hence, there is a case where creating an index futures whose payoff is the average of the insurers endowment risks mitigates the adverse selection enough for the index futures to be traded, although each insurer's individual insurance risk cannot be trade with the uninformed outside investor due to strong adverse selection.

Observe that if $N=1$, the condition $\Lambda_F>0$ is equivalent to $\Lambda>0$. Observe also that the larger N is, the weaker the restriction $\Lambda_F>0$ is. Thus, the futures of each insurer's individual risk is a special case of the average index futures, and as the number of the insurers in the average index increases, the index futures is more likely to be traded.

Proposition 1 also shows that for the mitigation of adverse selection by average-indexing to work, it is necessary that $\mathrm{Var}[x_c]>0$ i.e., a part of the insurer's endowment risk x_n should depend on the non-informational common market-wide risk x_c. This occurs because the payoff of the index futures is $F=\frac{1}{N}\Sigma_{n=1}^N x_n=\frac{1}{N}\Sigma_{n=1}^N(x_c+\varepsilon_n)=x_c+\frac{1}{N}\Sigma_{n=1}^N\varepsilon_n$ and hence averaging the payoffs x_n's reduces the ratio of risks $\frac{1}{N}\Sigma_{n=1}^N\varepsilon_n$ subject to asymmetric information in the total payoff F (in terms of variance) only if $\mathrm{Var}[x_c]>0$. Consequently, if the payoffs of insurance risks that the insurers are endowed are independent (i.e., $\mathrm{Var}[x_c]=0$ in this setup), averaging is useless to mitigate the adverse selection problem. Furthermore, this proposition implies that the larger (in terms of their variances) the portion of the non-informational common part x_c in the payoffs x_n's is, the more effective the averaging is for mitigating the adverse selection.

5. THE REINSURANCE MARKETS

Participants in the reinsurance markets are presumably professionals in insurance. They know equally well about insurance risks that they trade. At least, asymmetric information is much less among insurers and reinsurers than among insurers and non-insurance outside investors in the capital markets. Adverse selection problem about insurance risks is small in the reinsurance markets. However, the size of the reinsurance markets is much smaller than that of the capital markets. This limits the capacity of the reinsurance markets to bear the insurance risks, especially those associated with large natural disasters.

Motivated by this observation, we model the reinsurance markets as one representative reinsurer who has the same information as the insurers, but is less risk tolerant than the representative non-insurance outside investor. We consider the idealized reinsurance markets where the insurers and the reinsurer have the symmetric information, trade their risks competitively, and attain the optimal risk allocation.

In this idealized reinsurance markets, the insurance risks $x = (x_1, \dots, x_N)^\intercal$ of the insurers are traded. Denote by the $\theta_{R_n} = \left(\theta_{R_n 1}, \dots, \theta_{R_n N}\right)^\intercal$ the position that the insurer n takes in the reinsurance markets. Each insurer n trades the reinsurance contract with payoff x_n competitively. Let $P_R = (P_{R1}, \dots, P_{RN})^\intercal$ be the price of the reinsurance contracts. Then, the date 2 wealth of the insurer n after trading in the reinsurance markets is given by $W_{R_n} = e_n x_n + \theta_{R_n}^\intercal (x - P_R)$. It is more convenient to use the gross position $\phi_{R_n} \equiv \left(\theta_{R_n 1}, \dots, \theta_{R_n n} + e_n, \dots, \theta_{R_n N}\right)^\intercal$ for all n. In terms of the gross position, at date 1, the insurer n solves the following problem to obtain the optimal position ϕ_{R_n}:

$$\mathrm{Max}_{\phi_{R_n}} E[-\exp(-\frac{1}{\gamma_I} W_{R_n}) | S].$$

Its first order condition is

$$E[x|S] - P_R - \frac{1}{\gamma_I} \mathrm{Var}[x|S] \phi_n = 0. \tag{20}$$

and its second order condition is

$$\mathrm{Var}[x|S] > 0. \tag{21}$$

Since the second order condition is satisfied by assumption, the insurer n's optimal gross position $\phi_{R_n} \equiv \left(\phi_{R_n 1}, \dots, \phi_{R_n N}\right)^{\top}$ is given by

$$\phi_{R_n} = \gamma_I \mathrm{Var}[x|S]^{-1}\{E[x|S] - P_R\}. \tag{23}$$

Denote by $\phi_{RE} \equiv (\phi_{RE1}, \dots, \phi_{REN})^{\top}$ the position that the representative reinsurer takes. The reinsurer has the same information as the insurers and trades the reinsurance contracts competitively. Then, the reinsurer's date 2 wealth after trading the reinsurance contracts is given by $W_{RE} = \phi_{RE}^{\top}(x - P_R)$. At date 1, the reinsurer solves the following problem to obtain the optimal position θ_{RE}:

$$\mathrm{Max}_{\phi_{RE}} E[-\exp(-\frac{1}{\gamma_{RE}} W_{RE})|S].$$

Its first order condition is

$$E[x|S] - P_R - \frac{1}{\gamma_{RE}} \mathrm{Var}[x|S]\phi_{RE} = 0. \tag{24}$$

and its second order condition is

$$\mathrm{Var}[x|S] > 0. \tag{25}$$

Since the second order condition is satisfied by assumption, the insurer n's optimal gross position $\phi_{RE} \equiv (\phi_{RE1}, \dots, \phi_{REN})^{\top}$ is given by

$$\phi_{RE} = \gamma_{RE} \mathrm{Var}[x|S]^{-1}\{E[x|S] - P_R\}. \tag{26}$$

Let $e \equiv \sum_{n=1}^{N} e_n$. In an equilibrium, $\sum_{n=1}^{N} \phi_{R_n} + \phi_{RE} = e$. From this market clearing condition, we obtain the equilibrium price P_R as follows:

$$P_R = E[x|S] - \frac{1}{N\gamma_I + \gamma_{RE}} \mathrm{Var}[x|S]e. \tag{27}$$

Hence, the investor n's equilibrium gross position is given by

$$\phi_{R_n} = \frac{\gamma_I}{N\gamma_I + \gamma_{RE}} e \tag{28}$$

and the reinsurer's equilibrium gross position is given by

$$\phi_{RE} = \frac{\gamma_{RE}}{N\gamma_I + \gamma_{RE}} e. \tag{29}$$

That is, in an equilibrium in the idealized reinsurance markets, the aggregated insurance risks e is allocated to the market participants according to their risk tolerance.

6. INSURER'S UTILITY IN TRADING THE INDEX FUTURES AND REINSURANCE CONTRACTS

We assume that the insurers trade either the index futures or the reinsurance contracts, i.e. do not trade both at the same time. Then, we compare the insurer's ex-ante utility levels between when the index futures is traded and when reinsurance contracts are traded to see the cases that the insurers choose to trade the index futures, rather than the reinsurance contracts, in order to share their insurance risks with the non-insurance outside investors i.e., the capital markets.

The insurers' ex-ante utility when they trade the index futures is given as follows:

Lemma 3:

When the index futures is traded, the ex-ante utility of the insurer n is given by

$$E[u_I(W_{F_n}) = -|I_3 + \frac{2}{\gamma_I}\Sigma_F A_F|^{-1/2}$$

where

$$\Sigma_F \equiv \begin{bmatrix} Var[s_n] & 0 & \alpha_0 Var[s_n] \\ 0 & Var[e_n] & -\alpha_2 Var[e_n] \\ \alpha_0 Var[s_n] & -\alpha_2 Var[e_n] & Var[\theta_{F_n}] \end{bmatrix}$$

$$A_F \equiv \begin{bmatrix} 0 & \frac{Cov[x_n, s_n]}{2Var[s_n]} & 0 \\ \frac{Cov[x_n, s_n]}{2Var[s_n]} & \frac{-1}{2\gamma_I} Var[x_n|s_n] & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

and

$$\alpha_0 \equiv \frac{1}{(N+1)k_F + \frac{1}{\gamma_I} Var[F|S]} \cdot \frac{Cov[x_n, s_n]}{Var[s_n]} \cdot \frac{1}{N},$$

$$\alpha_1 \equiv \frac{1}{k_F + \frac{1}{\gamma_I} Var[F|S]} \cdot \frac{k_F}{(N+1)k_F + \frac{1}{\gamma_I} Var[F|S]} \cdot \frac{1}{\gamma_I} Cov[F, x_n|S]$$

$$\alpha_2 \equiv \frac{1}{k_F + \frac{1}{\gamma_I} Var[F|S]} \cdot \frac{Nk_F + \frac{1}{\gamma_I} Var[F|S]}{(N+1)k_F + \frac{1}{\gamma_I} Var[F|S]} \cdot \frac{1}{\gamma_I} Cov[F, x_n|S]$$

$$\alpha_3 \equiv k_F + \frac{1}{2\gamma_I}\mathrm{Var}[F|S]$$

$$\theta_n \equiv \alpha_0 \Sigma_{m=1}^{N} s_m + \alpha_1 \Sigma_{m\neq n}^{N} e_m - \alpha_2 e_n,$$

$$\mathrm{Var}\left[\theta_{F_n}\right] = N\alpha_0^2 \mathrm{Var}[s_n] + (N-1)\alpha_1^2 \mathrm{Var}[e_n] + \alpha_2^2 \mathrm{Var}[e_n].$$

Meanwhile, the insurers' ex-ante utility when they trade the reinsurance contracts is given as follows:

Lemma 4:

When the reinsurance contracts are traded, the ex-ante utility of the insurer n is the same as the insurer 1's and is given by

$$E[u_I(W_{R_n}) = -|I_{N+1} + \frac{2}{\gamma_I}\Sigma_{RE}A_{RE}|^{-1/2}$$

where

$$\Sigma_{RE} \equiv \begin{bmatrix} \mathrm{Var}[s_1] & 0 & 0 & 0 & 0 \\ 0 & \mathrm{Var}[e_1] & 0 & 0 & 0 \\ 0 & 0 & \mathrm{Var}[e_2] & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathrm{Var}[e_N] \end{bmatrix}$$

$$A_{RE} \equiv \begin{bmatrix} 0 & \frac{\mathrm{Cov}[x_1, s_1]}{2\mathrm{Var}[s_1]} & \beta_{1,3} & \cdots & \beta_{1,N+1} \\ \frac{\mathrm{Cov}[x_1, s_1]}{2\mathrm{Var}[s_1]} & \beta_{2,2} & & & \\ \beta_{3,1} & & \ddots & & \\ \vdots & & & \ddots & \\ \beta_{N+1,1} & & & & \beta_{N+1,N+1} \end{bmatrix}$$

and

$$\beta_{2,2} = \frac{1}{2\gamma_I}[\{-1 - \frac{2\gamma_I}{N\gamma_I + \gamma_{RE}} + \left(\frac{\gamma_I}{N\gamma_I + \gamma_{RE}}\right)^2\}\mathrm{Var}[x_1|s_1]$$
$$+\mathrm{Cov}[x, x_1|S]^{\mathsf{T}}\mathrm{Var}[x|S]^{-1}\mathrm{Cov}[x, x_1|S]$$

$$\beta_{1,n} = \beta_{n,1} = 0$$

$$\beta_{n,n} = \frac{1}{2\gamma_I}\left(\frac{\gamma_I}{N\gamma_I + \gamma_{RE}}\right)^2 \mathrm{Var}[x_n|s_n] \quad \text{for } n = 3, \dots, N+1.$$

$$\beta_{2,n} = \beta_{n,2} = \frac{1}{4\gamma_I}\{\frac{-2\gamma_I}{N\gamma_I + \gamma_{RE}} + \left(\frac{\gamma_I}{N\gamma_I + \gamma_{RE}}\right)^2\}\mathrm{Cov}[x_1, x_n|S] \quad \text{for n=3,…, N+1}$$

$$\beta_{m,n} = \beta_{n,m} = \frac{1}{4\gamma_I}\left(\frac{\gamma_I}{N\gamma_I+\gamma_{RE}}\right)^2 \mathrm{Cov}[x_1, x_n|S]$$

for m, n= 3,…, N+1, $m \neq n$

By comparing both utility levels, we find when the insurers choose the index futures.

Proposition 2:

The insurers wish to trade the index futures rather than the reinsurance contracts, if and only if

$$E[u_I(W_{F_n})] > E[u_I(W_{R_n})]$$

where $E[u_I(W_{F_n})]$ and $E[u_I(W_{R_n})]$ are given by Lemma 3 and 4, respectively.

7. NUMERICAL EXAMPLES

The following numerical examples describe the situation discussed above. We set $\mathrm{Var}[\varepsilon_n] = \mathrm{Var}[s_n] = 1$, $\mathrm{Var}[e_n] = 5$, $\mathrm{Cov}[\varepsilon_n, s_n] = 0.6$, $\gamma_I = \gamma_u = 10$, and $\gamma_{RE} = 5$. We also set $\mathrm{Var}[x_c] = 1$ unless we investigate the effect of varying $\mathrm{Var}[x_c]$. Note that under these parameter values, Λ in equation (8) is strictly negative. That is, $\Lambda < 0$ and each insurer n cannot trade his own endowed insurance risk individually with the non-insurance outside investor because of the strong adverse selection.

Figure 1 shows how the value of Λ_F in equation (18) varies as the number of insurers N varies from 1 to 10. Denote by $\Lambda_F(N)$ the value of Λ_F when the number of insurers (or the number of the insurers' risks included in the index futures) is N. As we have already discussed after Proposition 1, $\Lambda_F(1) = \Lambda$. Hence, $\Lambda_F(1) < 0$ i.e., the futures cannot be traded due to strong adverse selection when the number of insurers is 1. Note also that $\Lambda_F(2) < 0$ i.e., the index futures cannot be traded when the number of insurers is 2, either.[4] In general, however, the larger N is, the less is the adverse selection in trading the index futures because of the diversification of the insurers' specific risks, which makes it easier for the informed insurers to trade the index futures with the uninformed outside investors. Consequently, when N is larger than or equal to 3 in this example, $\Lambda_F(N) > 0$ and the index futures can be traded between the insurers and the investor.

[4] Under the assumed parameter values, $\Lambda_F(1) = \Lambda = -0.226$ and $\Lambda_F(2) = -0.012$.

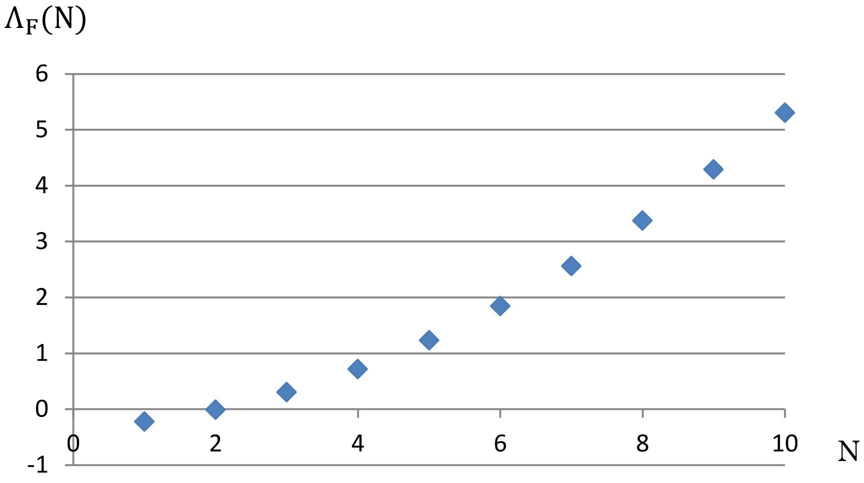


Figure 1: Values of Λ_F when the number of insurers is N.

Proposition 1 also implies that it is necessary that $Var[x_c] > 0$ for the mitigation of adverse selection by average-indexing to work and that the larger (in terms of their variances) the portion of the non-informational common market-wide risk x_c in the payoffs x_n's is, the more effective the averaging is for mitigating the adverse selection. Figure 2 plots the minimum number N_{min} of insurers in the index futures that is necessary for the futures to be traded. The value of $Var[x_c]$ varies from 0.1 to 1.5. Clearly, N_{min} decreases, as $Var[x_c]$ increases. ($N_{min} = 21$ when $Var[x_c] = 0.1$ and $N_{min} = 2$ if $Var[x_c] \geq 1.1$ in this example.)

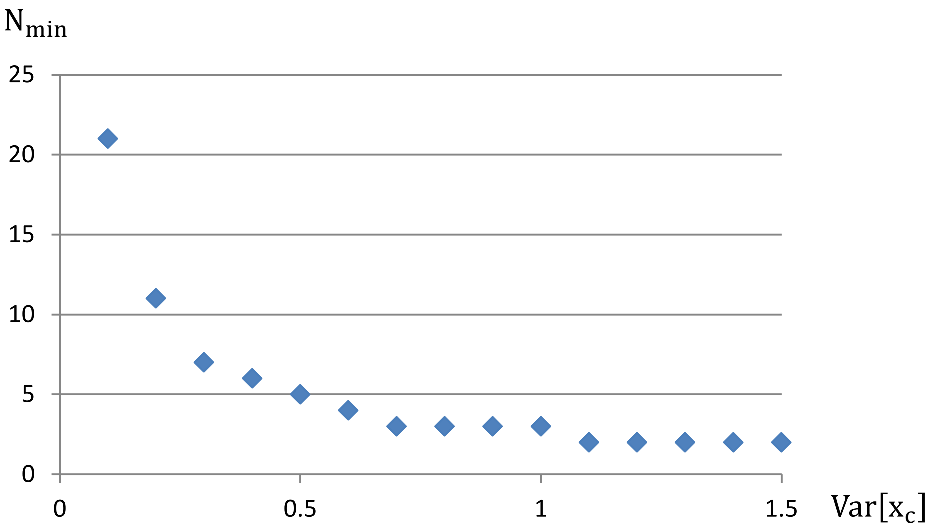
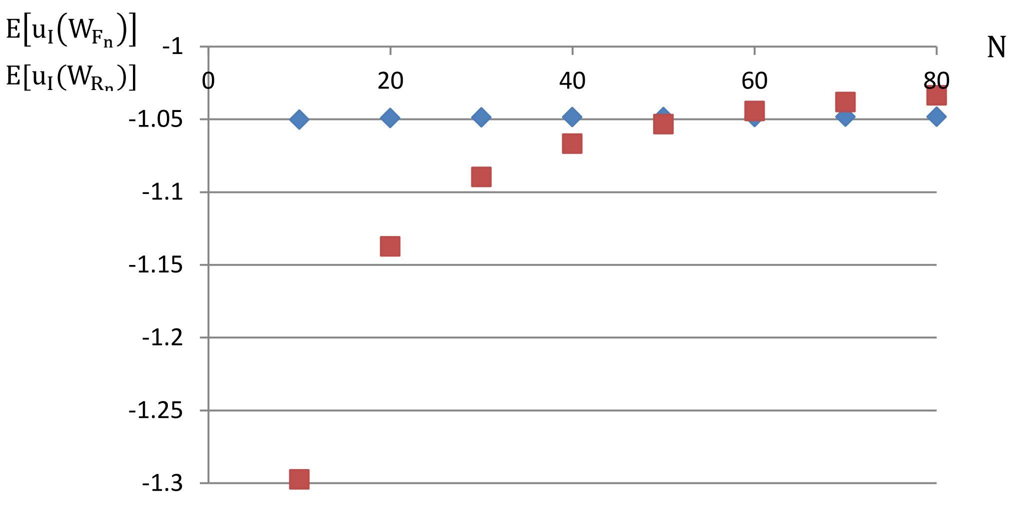


Figure 2: The minimum number N_{min} of insurers needed to make the index futures tradable when the variance of common market-wide risk is $Var[x_c]$.

Figure 3 compares the insurer n's utility $E[u_I(W_{F_n})]$ when he trades the index futures and the utility $E[u_I(W_{R_n})]$ when he shares risks through the reinsurance markets for the different number N of insurers included in the index futures. In general, the larger N is, the less is the adverse selection in trading the index futures because of the reduction of asymmetric information through diversification. This makes it easier for the insurers to trade the index futures with the uninformed outside investors. However, this also increases the basis risks that the insurers face in trading the futures. The benefit of trading the index futures is determined by the trade-off between the decrease of the adverse selection and the increase of the basis risks. In this example, the former is more effective than the latter so that the utility of the insurer increases as N increases.

For the insurers, whether to use the CAT index futures or the reinsurance markets depends on which gives them the better hedging opportunities. Note that we assume that there is neither asymmetric information nor basis risk for the insurers in the reinsurance markets. In this example, despite the adverse selection, the index futures provides better hedging opportunity to the insurers when the number N of the insurers is small because of the larger risk tolerance of the outside investor.[5] As N increases, however, the basis risk of the index futures becomes larger and the reinsurance markets without basis risk becomes more attractive than the index futures to the insurers.



[5] Note that in this setup, as the number N of the insurers increases, the number of the participants in the futures and reinsurance markets also increases. Since the calculated utility levels are affected by the change of the number of market participants, we need to be careful in interpreting the effects of increasing N. Note also that the insurers behave differently in the futures markets and the reinsurance markets: They trade the index futures strategically while they trade the reinsurance contracts competitively.

Figure 3: Utility levels as N increases where the rhombate dots indicates the utility $E[u_I(W_{F_n})]$ when he trades the index futures and the square dots indicates the insurer n's utility $E[u_I(W_{R_n})]$ when he shares risks through the reinsurance markets.

8. CONCLUSION

We have investigated the conditions under which an average index futures of several insurance risks can be traded in the presence of asymmetric information. We have found that even when each insurer's individual risk cannot be traded due to strong adverse selection, averaging such risks can mitigate the adverse selection enough so that the index futures with the average payoff can be traded, if the number of the risks included in averaging is large enough.

On the other hand, we have also found that averaging may reduce the attractiveness of the index futures due to the increase of basis risk. Consequently, the insurers may prefer sharing risks through reinsurance contracts that have no basis risk rather than trading the index futures, although the risk tolerance of reinsurance markets is much smaller than that of the futures markets. This may be interpreted as a part of the reason why the CAT index futures, with payoff being industry average of insurers' losses, have been so unsuccessful, despite its clear theoretical importance as a new tool to transfer insurance risks to the capital markets under adverse selection.

There are several points worth mentioning for future research interests. First, in the analysis above, we have assumed that each insurer's risk cannot be traded individually with the uninformed outside investor due to strong adverse selection. This assumption implies, however, that CAT bonds that trade insurers' individual insurance risks without basis risk cannot be traded. Thus, CAT index futures should be more likely to be traded than CAT bonds in our setup. Clearly, this does not fit the fact that CAT bonds are much more popular than CAT index futures. To understand the success of CAT bonds, we may need to relax this assumption and compare the effects of basis risk and adverse selection in a broader situation.

Second, we have investigated only the cases where the insurers trade either the index futures with the outside non-insurance investor or the reinsurance contracts with the reinsurer. Furthermore, the reinsurance markets are idealized in that the insurers and the reinsurer can trade all reinsurance contracts without friction. Examining the case where the insurers can trade only the limited kinds of reinsurance contracts is a possible extension of the analysis.

Finally, though we have just compared the use of the capital markets and that of the reinsurance markets from the insurers' viewpoint, they indeed interact with each other. Allowing the reinsurer to trade both the reinsurance contracts and the index futures at the same time may clarify the role of the reinsurer as an intermediary of the insurance risks among several markets. This is an immediate task in further research.

APPENDIX :

Math. Lemma 1:

Let I_N be the $N \times N$ identity matrix, J_N be the $N \times N$ matrix with all elements equal to unity, and a and b be scalars where $a \neq 0$. Then,

$$(aI_N + bJ_N)^{-1} = \frac{1}{a}\left(I_N - \frac{b}{a+bN} J_N\right)$$

Proof of Math. Lemma 1: Direct computation.‖

Math. Lemma 2:

Let A be a symmetric $N \times N$ matrix, B be an $N \times 1$ vector, and C be a scalar. Suppose that $e \sim N(0, \Sigma)$. Then, if and only if $I_N - 2\Sigma A$ is positive definite,

$E[\exp(e^\intercal Ae + B^\intercal e + C)] = |I_N - 2\Sigma A|^{-1/2} \exp(B^\intercal (I_N - 2\Sigma A)^{-1}\Sigma B + C)$.

Proof of Math. Lemma 2:

$$
\begin{aligned}
&E[\exp(e^\intercal Ae + B^\intercal e + C)] \\
&= \int_{R^N} \exp(e^\intercal Ae + B^\intercal e + C)\,(2\pi)^{-\frac{N}{2}}\,|\Sigma|^{-\frac{1}{2}} \exp\left(\frac{-1}{2} e^\intercal \Sigma e\right) de \\
&= \int_{R^N} (2\pi)^{-\frac{N}{2}} |\Sigma|^{-\frac{1}{2}} \exp\left(\frac{-1}{2}(e-H)^\intercal(\Sigma^{-1} - 2A)(e-H)\right. \\
&\qquad\qquad \left. + \frac{1}{2} B^\intercal (\Sigma^{-1} - 2A)^{-1} B + C\right) de \\
&= |\Sigma|^{-1/2}\, |(\Sigma^{-1} - 2A)^{-1}|^{-1/2} \exp(B^\intercal (\Sigma^{-1} - 2A)^{-1} B + C) \\
&= |I_N - 2\Sigma A|^{-1/2} \exp(B^\intercal (I_N - 2\Sigma A)^{-1} \Sigma B + C)
\end{aligned}
$$

where $H \equiv (\Sigma^{-1} - 2A)^{-1}B$. ‖

Proof of Lemma 1:

Since $\theta + \theta_u = 0$ in equilibrium, we have $H = 0$. Then, by equation (1) and Math. Lemma 1, the investor can obtain $\theta = (\theta_1, \dots, \theta_N)^\intercal$ by observing $P = (P_1, \dots, P_N)^\intercal$ if $k \neq 0$ and $k \neq Nl$. Moreover, by equation (4), the investor can recover $Q \equiv (q_1, \dots, q_N)^\intercal$ if $2k + \frac{1}{\gamma_I} Var[x_n|S] - l \neq 0$ and $2k + \frac{1}{\gamma_I} Var[x_n|S] - l + N(l - 1) \neq 0$. Assuming these conditions are met,[6] we have

[6] When some of these conditions are not satisfied, we can find small perturbation of the relevant parameter values that makes the conditions to be met.

$$k = \frac{1}{1-2\frac{\mathrm{Cov}[x_n,q_n]}{\mathrm{Var}[q_n]}}\{\frac{\mathrm{Cox}[x_n,q_n]}{\mathrm{Var}[q_n]}\frac{1}{\gamma_I}\mathrm{Var}[x_n|s_n] + \frac{1}{\gamma_u}\mathrm{Var}[x_n|q_n]\},\text{ and}$$

$$l = \frac{1}{1-2\frac{\mathrm{Cov}[x_n,q_n]}{\mathrm{Var}[q_n]}}\frac{1}{\gamma_u}\mathrm{Var}[x_n].$$

Substituting them, the second order condition of the insurers will be

$$\frac{1}{1-2\frac{\mathrm{Cov}[x_n,q_n]}{\mathrm{Var}[q_n]}}\left\{\frac{1}{\gamma_I}\mathrm{Var}[x_n|s_n] + \frac{2}{\gamma_u}\mathrm{Var}[x_n|q_n]\right\} > 0,$$

which is satisfied if and if

$$1 - 2\frac{\mathrm{Cov}[x_n,q_n]}{\mathrm{Var}[q_n]} > 0.$$

This is equivalent to

$$\frac{1}{\gamma_I^2}\mathrm{Var}^2[x_n|s_n]\mathrm{Var}[e_n] - 2\frac{\mathrm{Cov}^2[x_n,s_n]}{\mathrm{Var}[s_n]} > 0. \ \|$$

Proof of Lemma 2: Similar to the proof of Lemma 1. ||

Proof of Proposition 1: Clear from Lemma 1 and 2. ||

Proof of Lemma 3:

Direct calculation shows that the insurer's utility $\mathrm{E}\left[u_I\left(W_{F_n}\right)\right]$ is given by $\mathrm{E}\left[u_I\left(W_{F_n}\right)\right] = \mathrm{E}[-\exp\left(\frac{-1}{\gamma_I}\xi^\intercal A_F\xi\right)]$

where $\xi \equiv (s_n, e_n, \theta_n)^\intercal$, $\theta_n \equiv \alpha_0\Sigma_{m=1}^N s_m + \alpha_1\Sigma_{m\neq n}^N e_m - \alpha_2 e_n$,

$$A_F \equiv \begin{bmatrix} 0 & \dfrac{\mathrm{Cov}[x_n,s_n]}{2\mathrm{Var}[s_n]} & 0 \\ \dfrac{\mathrm{Cov}[x_n,s_n]}{2\mathrm{Var}[s_n]} & \dfrac{-1}{2\gamma_I}\mathrm{Var}[x_n|s_n] & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

and $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ are given in Lemma 3. Applying Math. Lemma 2, we obtain the desired result. ||

Proof of Lemma 4: Similar to the proof of Lemma 3. ||

Proof of Proposition 2: Clear from Lemma 3 and 4. ||

REFERENCES

[1] Allen, F. and Gale, D., "Financial Innovation and Risk Sharing," MIT Press, Cambridge MA, USA, 1994.

[2] Bhattacharya, U., Reny, P. J., and Spiegel, M., "Destructive Interference in an Imperfectly Competitive Multi-Security Market," Journal of Economic Theory, Vol.65, pp.136-170, 1995.

[3] Bhattacharya, U. and Spiegel, M., "Insiders, Outsiders, and Market Breakdowns," Review of Financial Studies Vol.4, pp.255-282, 1991.

[4] Barrieu, P. and Louberge, H., "Hybrid Cat-Bonds," Journal of Risk and Insurance Vol.76, pp.547-578, 2009.

[5] Cummins, J. D. and Barrieu, P., "Innovation in Insurance Markets: Hybrid and Securitized Risk-Transfer Solutions." in *Handbook of Insurance* (Dionne, G. ed.); DOI: 10.1007/978-1-4614-0155-1_20, 2013.

[6] Cummins, J. D., Lalonde, D., and Phillips R. D., "The Basis Risk of Index-Linked Catastrophe Loss Securities," Journal of Financial Economics Vol.71, pp.77-111, 2004.

[7] DeMarzo, P., and Duffie, D., "A Liquidity-Based Model of Security Design," Econometrica 67, 65-90, 1999.

[8] Doherty, N., A., "Innovations in Managing Catastrophe Risk," Journal of Risk and Insurance Vol.64, pp.713-718, 1997.

[9] Doherty, N., A., and Richter, A., "Moral Hazard, Basis Risk, and Gap Insurance," Journal of Risk and Insurance Vol.69, pp.9-24, 2002.

[10] Duffie, D., and Rahi, R., "Financial Innovation and Security Design," Journal of Economic Theory Vol.65, pp.1-42, 1995.

[11] Ganapati, S., Retik, M., Puleo, P., and Starr, B., "Catastrophe-Linked Securities," Chapter 20 in *Handbook of Structured Financial Products* (Fabozzi, F. ed.), Frank J.

Fabozzi Associates, New Hope, Penn- sylvania, USA, 1998.

[12] Gibson, R., Habib, M. A., Ziegler, A., “Reinsurance or Securitization: The case of Natural Catastrophe Risk,” Journal of Mathematical Economics Vol.53, pp.79-100, 2014.

[13] Guy Carpenter, Catastrophe Bond Update: Fourth Quarter and Full Year 2015, 2015.

[14] Harrington, S., E., and Niehaus, G., “Basis Risk with PCS Chatastrophe Insurance Derivative Contracts,” Journal of Risk and Insurance Vol. 66, pp.365-389, 2003.

[15] Kist, F. O., and Meyers, G., “Evaluating the effectiveness of index- based insurance derivatives in hedging property/casualty insurance transaction,” American Academy of Actuaries Index Securitization Task Force, 1999.

[16] Major, J. A., “Index Hedge Performance: Insurer Market Penetration and Basis Risk,” Chapter 10 in *The Financing of Catastrophe Risk*, (K. A. Froot, K. A. ed.) The University of Chicage Press, Chicago, Illinois, USA, 1999.

[17] Marin, J. M., and Rahi, R., “Speculative Securities,” Economic Theory Vol.14, pp.653-668, 1999.

[18] Nell, M. and Richter, A., “Improving Risk Allocation through Indexed CAT Bonds,” The Geneva Papers on Risk and Insurance Vol.29, pp.183-201, 2004.

[19] Ohashi, K., “Security Innovation on Several Assets under Asymmetric Information,” Japanese Economic Review Vol.50, pp.75-95, 1999.

[20] Ohashi, K., “When Should a CAT index Futures be Created?” Discussion Paper No. 576, The Institute of Social and Economic Research, Osaka University, 2003.

[21] Rahi, R., “Adverse Selection and Security Design,” Review of Economic Studies Vol.63, pp.287-300, 1996.

[22] Subrahmanyam, A., “A Theory of Trading in Stock Index Futures,” Review of

Financial Studies Vol.4, pp.17-51, 1991.

[23] Subramanian, A. and Wang, J. , “Catastrophe Risk Transfer,” Working Paper, Georgia State University, 2014.