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A factor model of random thinning for top-down type credit portfolio risk assessment [*1]

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Abstract

In this paper, we propose a new random thinning model in a so-called top-down framework so as to improve credit risk assessment of some sub-portfolios. Specifically we extend a model of thinning process adjusted with credit quality vulnerability factors (CQVF) introduced in Yamanaka et al. [6] so that CQVF can be specified in terms of some time varying observable factors and that the thinning process can be regarded as a factor model. Some empirical studies on the proposed thinning process with historical data of credit rating transitions of Japanese firms show that both "alpha" and "beta" estimated from the TOPIX sector indices are significant for some sub-portfolios classified by industry. Also, we try a numerical examination to see that our model can be used for practical credit risk assessment.

1 Introduction

In this paper, we propose a new random thinning model in a so-called top-down framework so as to improve credit risk assessment of some sub-portfolios. Specifically we extend a model of credit quality vulnerability factor (CQVF) regarded as a stability measurement of credit quality of each sub-portfolio introduced in Yamanaka et al. [6] so that CQVF can be specified in terms of some time varying observable factors and that the thinning process can be regarded as a factor model.

Financial institutions have been required to highly develop their risk management by regulation. Accordingly, as for credit risk management, they have to comprehend how the risks are distributed and related among some credit sub-portfolios as well as quantify the total risk of the whole credit portfolio.

We use an intensity based credit risk model in a so-called top-down approach so as to investigate the credit risk dependence of credit sub-portfolios. It is said that the top-down approach has an advantage to allow a relatively simple representation of credit risk dependence among constituents of a large portfolio.

In general, modeling within the top-down approach has a couple of steps, "top" part and "down" part. Firstly ("top" part), we specify the intensity model associated with

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the counting process of target credit events occurred in the universe portfolio, where we ignore the constituents of the portfolio. Secondly ("down" part), we obtain the intensity models corresponding to segmented sub-portfolios of the universe portfolio by allotting the original intensity model to sub-portfolios via random thinning.

As for random thinning, several methods have been proposed in previous works as follows. Giesecke et al. [1] and Halperin and Tomecek [3] proposed piece-wise constant thinning model for calculating portfolio constituent firms risk contribution. In addition to these, the thinning models which references default probabilities based on stock values are proposed by Kunisch and Uhrig-Homburg [5] and Kaneko and Nakagawa [4]. Giesecke and Kim [2] and Yamanaka et al. [8] considered the thinning models for dissecting the portfolio risk into sub-portfolio contributions, which are specified by the number of portfolio constituents. In those previous works, anyway the main idea of thinning is that the original whole intensity is proportionally allotted according the the size of each sub-portfolio.

In order to correct the way of allotting the original whole intensity into sub-portfolios, Yamanaka et al. [6] introduced a parameter called "credit quality vulnerability factor (CQVF)." Yamanaka et al. [6] assumed a time invariant CQVF model for their empirical analysis and concluded that the CQVF indicates the existence of some factors for the credit event frequency of each sub-portfolio, which is different from portfolio size effect.

However, credit risk factors specific to each sub-portfolio can change in time since business conditions are rapidly changing. In short it is natural to suppose that the CQVF can vary in time as Yamanaka et al. [6] pointed out. As such, Yamanaka et al. [7] suggested a simple piece-wise latent factor model in order to capture dynamics of CQVF values.

In this paper, we propose a new model of time varying CQVF in a different way from Yamanaka et al. [7]. Specifically we introduce a CQVF model that can be specified in terms of some time varying observable factors, so the thinning process can be regarded as a factor model of CQVF.

Then, we show some results of empirical analysis on estimating our new CQVF model of the credit rating transition events for industrial sector portfolios with the historical credit rating transitions reported in Japan. Especially, we employ some explanatory variables obtained from sector-based stock indices and examine significance of such variables in our empirical analysis. In addition, we try a numerical examination to see if our model is applicable to credit risk assessment. The consequences imply a positive answer.

This paper is organized as follows. Section 2 briefly reviews a general formulation of top-down approach and explains a specification of our random thinning model with CQCF with some observable variables. Section 3 shows some empirical results. Section 4 concludes.

2 Top-down approach

In this section, we give a quick review of an intensity based credit risk model in the top-down approach and provide a random thinning model with a CQVF specified by time varying observable variables.

2.1 Intensity model in the top-down approach

Let us consider the continuous time $[0,\infty)$ and introduce a filtered complete probability space $(\Omega,\ \mathcal{F},\ \mathbb{P},\ \{\mathcal{F}_t\}_{t\geq 0})$, where $\{\mathcal{F}_t\}_{t\geq 0}$ is a right-continuous and complete filtration.

Denote by $\bar{S}$ the set of all debtors, called the universe credit portfolio and let I be an integer more than one. We suppose that this universe credit portfolio is decomposed into non-empty sub-portfolios $\{S_i\}_{i=1,\cdots,I}$, in short, we have $S_1 \cup \cdots \cup S_I = \bar{S}$ and $S_i \cap S_j = \emptyset$ $(i \neq j)$.

Next we introduce a marked point process $(\{T_n,\ s(n)\})$ whose pair consists of the times when target each credit event happens and the debtor corresponding to the event.

More specifically, let $\mathcal{T} = \{T_n\}_{n=1,2,\cdots}$ be a strictly increasing sequence of totally inaccessible $\{\mathcal{F}_t\}$-stopping times with $0 < T_1 < T_2 < \cdots$. We regard the stopping times $\{T_n\}_{n=1,2,\cdots}$ as the ordered credit event times observed in the whole credit portfolio $\bar{S}$. Let $s(n) \in \bar{S}$ be the debtor where the n-th credit event occurs.

We denote by $N_t = \sum_{n\geq 1} \mathbf{1}_{\{T_n \leq t\}}$ the counting process of credit events observed in the whole credit portfolio $\bar{S}$, and by λ_t the associated intensity process, which is indeed an $\{\mathcal{F}_t\}$-progressively measurable non-negative process such that the compensated process $N_t - \int_0^t \lambda_s \mathrm{d}s$ is an $\{\mathcal{F}_t\}$-local martingale. Often λ_t is modeled by a stochastic intensity process of Hawkes process, that is, a counting process with a self-exciting property. In this study, we do not specify the model of λ_t since we pay attention to modeling of random thinning explained later, which can be discussed independently of the universe-portfolio intensity process λ_t.

Accordingly, the counting process of credit events in sub-portfolio S_i is given by $N_t^i = \sum_{n\geq 1} \mathbf{1}_{\{T_n \leq t,\ s(n)\in S_i\}}$. Then the intensity λ_t^i associated with the the sub-portfolio S_i is obtained by using the procedure called a random thinning.

Let $\{Z_t^i\}_{i=1,\cdots,I}$ be $[0,\ 1]$-valued $\{\mathcal{F}_t\}$-adapted processes called thinning processes such that $\sum_{i=1}^{I} Z_t^i = 1, {}^{\forall}t \geq 0$.

For each $i = 1,\cdots,I$, with the thinning process Z_t^i, it follows that the intensity process λ_t^i for sub-portfolio S_i can be given by

$$\lambda_t^i = Z_t^i \lambda_t^*. \tag{1}$$

2.2 Specification of thinning processes

We introduce our model of random thinning processes $\{Z_t^i\}_{i=1,\dots,I}$ as in (2) below. They are specified by the proportion of the sub-portfolio size weighted by positive $\{\mathcal{F}_t\}$-adapted processes $\{\theta_t^i\}_{i=1,\dots,I}$, which Yamanaka et al. [6] intoroduced as"Credit Quality Vulnerability Factor (CQVF)" .

The CQVF can be interpreted as some factors that affect credit event frequency in a different way from the portfolio size effect. As θ_t^i is larger, the credit event of sub-portfolio S_i is likely to be more frequent, that is the credit quality of S_i would be more vulnerable. On the other hand, as θ_t^i is smaller, the credit events in S_i is likely to occur less frequently, that is the credit quality of S_i would be less vulnerable.

$$Z_t^i = \frac{\theta_t^i |S_i|}{\sum_i \theta_t^i |S_i|} \text{ for } i = 1, \dots, I, \tag{2}$$

where $|S_i|$ denotes the number of elements in S_i.

The number of debtors in sub-portfolio can increase due to new debtors' arrival or decrease due to complete payment without refinancing and default of debtors. We assume that S_i do not varies, namely our thinning model is formulated by (2) for simplicity since it does not matter with empirical analysis as we see below[*2]. However we can also consider the extended model which admits a time varying case of the cardinality of sub-portfolio.

In this paper, we specify the CQVF processes $\{\theta_t^i\}_{i=1,\dots,I}$ in terms of some covariates which stand for some observable risk factors of the corresponding sub-portfolio S_i, specifically $\{\mathcal{F}_t\}$-adapted M-dimensional processes $\{Y_{m,t}^i\}_{m=1,\dots,M}$ as follows: for each $i = 1, \dots, I$,

$$\theta_t^i = \exp\left(a_1 Y_{1,t}^i + \cdots + a_M Y_{M,t}^i + \gamma^i\right), \tag{3}$$

where $\{a_m\}_{m=1,\dots,M}$ are common coefficients over all the sub-portfollios, and γ^i is a parameter viewed as a fixed effect for sub-portfolio S_i. We call this CQVF model as exponential regression CQVF model in this paper. In our empirical analysis, we estimate not only the model with fixed effect but also the model without fixed effect (by setting $\gamma^i = 0$).

2.3 Estimation Procedure

As indicated above. we can estimate the parameters in our exponential regression CQVF model (3) independently of the universe-portfolio intensity process λ_t.

Denote by $\Theta = (a_1, a_2, \dots, a_M, \gamma^1, \dots, \gamma^I)$ the model parameters to be estimated in (3).

[*2] In the empirical analysis below, we consider the Japanese large companies with credit rating given by an agency and they are segmented by industry. We consider only credit rating transitions and do not consider defaults.

The data for estimating the parameters Θ is a sample of pairs $\{(\tilde{T}_n, \tilde{s}(n))\}$ of the event times and the corresponding names and covariates $\{\tilde{Y}^i_{m,t}\}_{m=1,\ldots,M}$ observed in the sample period $[0, T]$.

We employ a maximum likelihood method for estimating Θ. Since $\{Z_{T_n}\}_{n=1,2,\cdots,N_t}$ are independent given the event times, the likelihood function is specified by

$$L(\Theta \mid \{(\tilde{T}_n,\ \tilde{s}(n))\}_{\tilde{T}_n \le T}, \{\tilde{Y}^i_{m,t}\}_{m=1,\ldots,M,0\le t\le T}) = \prod_{i=1}^{I} \prod_{\tilde{T}_n \le T | \tilde{s}(n) \in S_i} Z^i_{\tilde{T}_n}. \tag{4}$$

Hence, the log-likelihood function is described as follows:

$$\begin{aligned}
&\log L(\Theta \mid \{(\tilde{T}_n,\ \tilde{s}(n))\}_{\tilde{T}_n \le T}, \{\tilde{Y}^i_{m,t}\}_{m=1,\ldots,M,0\le t\le T}) \\
&= \sum_{i=1}^{I} \sum_{\tilde{T}_n \le T | \tilde{s}(n) \in S_i} \log\left(Z^i_{\tilde{T}_n}\right) \\
&= \sum_{i=1}^{I} \sum_{\tilde{T}_n \le T | \tilde{s}(n) \in S_i} \Big\{ \log(|S_i|) + \left(a_1 \tilde{Y}^i_{1,t} + \cdots + a_M \tilde{Y}^i_{M,t} + \gamma^i\right) \\
&\qquad\qquad - \log \sum_{i=1}^{I} \left(|S_i| \exp\left(a_1 \tilde{Y}^i_{1,t} + \cdots + a_M \tilde{Y}^i_{M,t} + \gamma^i\right)\right)\Big\}.
\end{aligned} \tag{5}$$

3 Empirical analysis

In this section, we illustrate some empirical results on our exponential regression CQVF model and demonstrate the applicability of the model to credit risk assessment. For the purpose, we classify a set of Japanese firms into 33 sub-portfolios via sector classification defined by the Tokyo Stock Exchange (TSE) so as to incorporate risk factors "alpha" and "beta" calculated from the sector stock index announced by TSE.

In addition, we examine momentum and/or reversal effects of rating changes by introducing dummy variables which indicates if a rating transition occurred during the last 6 months [*3] . At last, we compare the estimated distribution of the number of credit events with the realized number of credit events for each sub-portfolio, and examine if our model is useful for practical credit risk assessment.

3.1 Sample data

As sample data for estimation of the exponential regression CQVF model (3), we employ historical data on credit rating transition of Japanese firms. Especially, we

[*3] "Rating transition momentum" means the phenomenon that another downgrades (resp. upgrades) are likely to occur successively after one downgrade (resp. upgrade) occurrs. On the other hand, "revesal" means that a downgrade (resp. an upgrade) tends to occur after one upgrade (resp. dpwngrade) occurs.

use the historical records on downgrade and upgrade from April 1, 2000 to September 30, 2013 of the Japanese firms, which were announced by Rating and Investment Information, Inc. (R&I)*4.

We use the samples from April 1, 2000 to March 31, 2010 for parameter estimation as training data (in-sample), and the samples after March 31, 2010 as test data (out-of-sample). Thus we implicitly suppose that the whole credit portfolio consists of all the firms where R&I provided the credit rating each period.

We assume that the whole credit portfolio of the rated firms is classified into 33 sub-portfolios by industry according to "TSE 33 sector classification" shown in table 2 in the appendix A.

As for specification of exponential regression CQVF model (3), we use "alpha" and "beta" calculated from the data of 33 TOPIX sector indices respectively as the covariates $Y^i_{1,t}$ and $Y^i_{2,t}$ for sub-portfolio $i = 1, \dots, 33$ in (3). We tentatively employ such risk factors derived from the sector stock price indices. Hoewever we consider that there is some rationality in selecting them since it is highly probable that credit quality of each industry sector is more or less relevant to the stock market.

The method of calculating the risk factors"alpha"and "beta" is as follows.

First, in order to obtain "alpha" for each sector, we calculate both the annual return of TOPIX and the corresponding sector index for business days during the in-sample period. Then we calculate "alpha" $Y^i_{1,t}$ for sub-portfolio i as the time-series of the difference between the realized annual returns of TOPIX and the sector return:

$$Y^i_{1,t} = \text{annual } r^{\text{TOPIX}}_t - \text{annual } r^i_t, \tag{6}$$

where annual r^*_t is the realized annual return at time t of TOPIX while annual r^i_t the realized annual return at time t of i-th sector index*5.

Second, the "beta" $Y^i_{2,t}$ for each sector is obtained by the ratio of sample covariance over the last one year of daily TOPIX returns to sample covariance over one year between daily returns of TOPIX and the corresponding sector index, in short.

$$Y^i_{2,t} = \frac{\frac{1}{249}\sum_{k=0}^{249}\left(r^{\text{TOPIX}}_{t-k} - \frac{1}{250}\sum_{\ell=0}^{249} r^{\text{TOPIX}}_{t-\ell}\right)\left(r^i_{t-k} - \frac{1}{250}\sum_{\ell=0}^{249} r^i_{t-\ell}\right)}{\frac{1}{249}\sum_{k=0}^{249}\left(r^{\text{TOPIX}}_{t-k} - \frac{1}{250}\sum_{\ell=0}^{249} r^{\text{TOPIX}}_{t-\ell}\right)^2}, \tag{8}$$

where r^*_t is the actual daily return at time t of TOPIX while r^i_t the actual daily return at time t of i-th sector index

*4 Rating and Investment Information, Inc. (R&I) is one of the largest credit rating agencies in Japan.

*5 Actually we calculate the realized annual return at time t of an index as its 250 business day return, that is, we obtain annual return of the index denoted by S_t as

$$\frac{S_t - S_{t-250\text{business days}}}{S_{t-250\text{business days}}}, \tag{7}$$

where S_t and $S_{t-250\text{business days}}$ are the closing price of the index at the date t and 250 business days before date t respectively.

Also, we add the dummy variable indicating if an underlying event occurred during the last half year to the two risk factors so as to examine the existence of rating transition momentum and/or reversal effect. Let $Y^i_{3,t}$ and $Y^i_{4,t}$ be the dummy variables at time t resepectively for downgrades and upgrades during the last half year.

Hence the four risk factors are prepared as the candidates that may determine the dynamics of CQVF.

3.2 Estimation Results

We illustrate the estimation results for the exponential regression CQVF model with "alpha", "beta", and the two dummies below.

Table 1 shows the estimation results for selected models in the exponential regression CQVF model without fixed effects and with ones. These models are selected with AIC from all nested models with 12 combinations of factors. The values in parentheses are the standard errors. The parameter is judged significant if the absolute value of the estimate is larger than twice of the standard error (meaning about 5% significance level).

The estimated alpha in Table 1 is significantly negative (resp. positive) for any case. It indicates that the downgrades (resp. upgrades) is likely to happen more frequently for the sectors with low (resp. high) alpha. Similary, the results on the model without fixed effects shows the larger beta the more frequently both downgrades and upgrades are likely to happen. On the other hand, the beta for the exponential regression CQVF model with fixed effects is not significant, different from the cases without fixed effects. As for rating change momentum and/or reversal, the coefficients of the dummies are not significant for most cases although the estimates imply weak possibility of momentum effects.

The estimated fixed effect is significant for 14 sectors in downgrades and for 25 sectors in upgrades. Table 3 and Table4 in the appendix B show the estimated fixed effect per sector for the exponential regression CQVF modell with all candidate risk factors "alpha", "beta", two dummies for downgrades and upgrades during the last half year and the fixed effect. Such a result seems to indicate that there are some additional risk factors driving the CQVF other than "alpha", "beta' and the dummies.

For a comparison, we also estimate a constant model given by $\theta^i_t \equiv \exp\left(c^i\right)$ for a constant parameter c^i. The values of AIC of estimated constant models are 3180.0 for downgrades and 2716.7 for upgrades. It follows from this result and the results Table 1 that the exponential regression CQVF model is superior, with respect to AIC, to the constant model for both downgrades and upgrades.

3.3 Distribution of the number of rating transitions obtained from exponential regression CQVF model

In order to see if our exponential regression CQVF model can be useful for credit risk assessment, we achieve the (conditional) distribution at time T of the number

Table. 1: Estimates of the selected exponential regression CQVF model . The values in parentheses are the standard errors. LL and AIC stand for the value of log likelihood and Akaike's Information Criteria respectively.

	Down		Up	
	without fixed effect	with fixed effect	without fixed effect	with fixed effect
α	-0.689	-0.570	0.680	0.645
	(0.263)	(0.259)	(0.207)	(0.207)
β	0.347	-	0.735	-
	(0.154)	-	(0.178)	-
Down	-	-	-0.223	-0.286
	-	-	(0.117)	(0.116)
Up	-0.206	-0.162	-	-
	(0.108)	(0.106)	-	-
LL	-1598.1	-1553.3	-1353.0	-1318.4
AIC	3202.2	3176.6	2712.0	2706.9

of credit events that occur during some period $(T, T+\Delta]$ for some $\Delta > 0$ in i-th sub-portfolio, namely $P(N^i_{T+\Delta} - N^i_T \in \bullet \mid \mathcal{F}_T)$, from the estimated CQVF model with the training data on $[0, T]$ Then we compare the distribution with the observed number of downgrades or upgrades contained in the test data.

Remember that we use the samples from April 1, 2000 ($t = 0$) to March 31, 2010 ($t = T$) for estimation and those from January 1st, 2010 to December 30th, 2013 ($t = T + \Delta$) for testing.

From a viewpoint of AIC minimization, Table 1 indicates that the optimal exponential regression CQVF model is the model with fixed effect and using "alpha" and the dummy of upgrade (resp. downgrade) during the last half year as risk factors for downgrades (resp. upgrades).

The (conditional) distribution $P(N^i_{T+\Delta} - N^i_T \in \bullet \mid \mathcal{F}_T)$ for downgrades or upgrades in i-th sub-portfolio is achieved by Monte Carlo simulation. We try 100,000 random scenarios generated from the optimal exponential regression CQVF model .

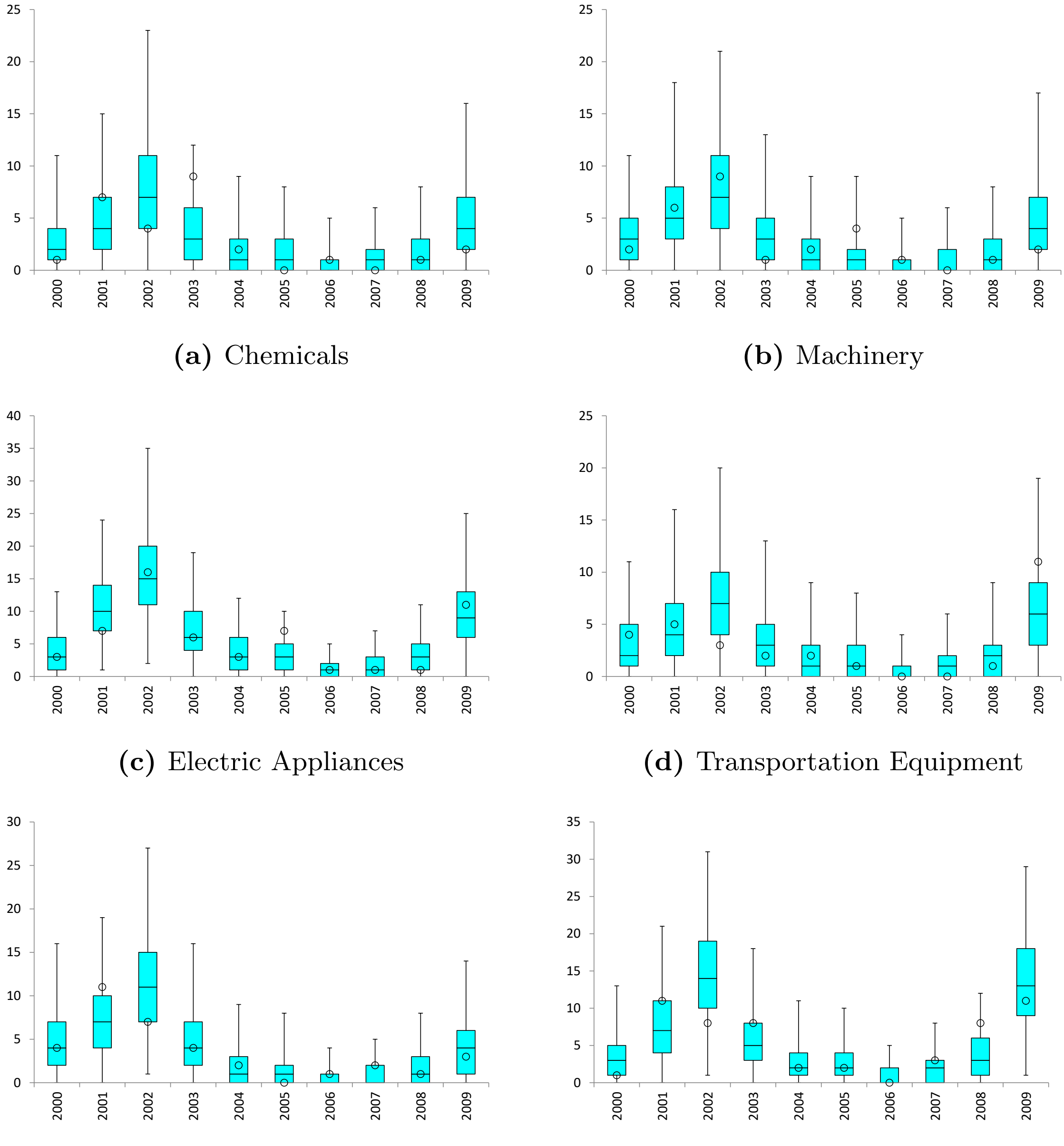
The simulation procedure is as follows:

i). Set the downgrade or upgrade times $\{T'_n\}$ such that $T < T'_1 < T'_2 < \cdots \leq T+\Delta$ in the whole test data.

ii). For each n, calculate $\{Z^i_{T'_n}\}_{i=1}^I$ with the optimal exponential regression CQVF model and allocate T'_n to one sub-portfolio randomly according to the allocation probability $\{Z^i_{T'_n}\}_{i=1}^I$. Each scenario, all the events in the test data are randomly allocated to sub-portfolios, regardless of which group the events actually belong to.

iii). Repeat the previous task 100,000 times and finally obtain the empirical distribution of the number of randomly allocated events for each sub portfolio.

Fig. 1 (resp. Fig. 2) displays the boxplots per sector of the distribution of the number of downgrades (resp. upgrades), which are obtained via Monte Carlo simulation mentioned above. Here, we selected the results of some big sectors in terms of the average size of the sector.

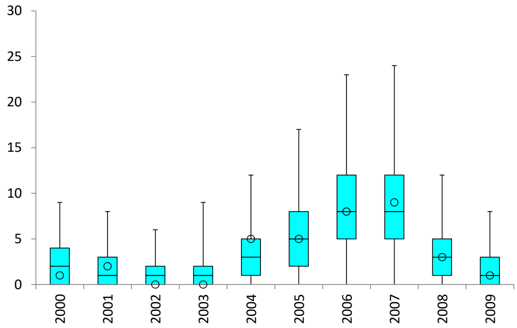


(a) Chemicals **(b)** Machinery

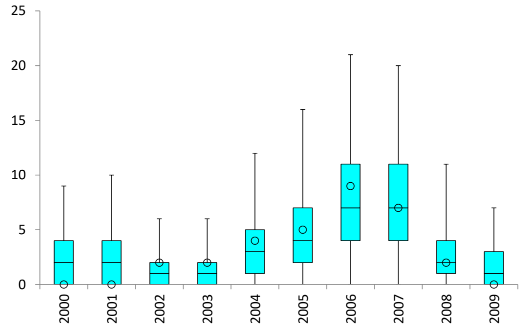
(c) Electric Appliances **(d)** Transportation Equipment

(e) Retail Trade **(f)** Banks

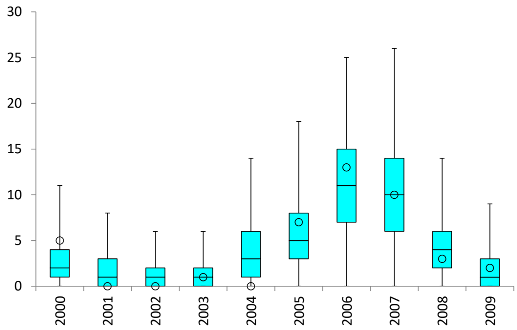
Fig. 1: The actual number of *downgrades* (circles) for *sector Chemicals, Machinery, Electric Appliances, Transportation Equipment, Retail Trade and Banks* in our sample data (from January 1st, 2000 to December 30th, 2009) as well as the boxplot per sector of the distribution of the number of downgrades during some period obtained via the simulation. As for the boxplot, the top and the bottom indicate the maximum and minimum respectively, the upper side and the lower side of each box indicate the 90th and the 10th percentile respectively, and the band inside does the median.



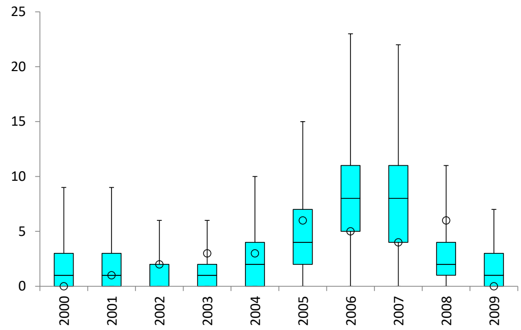
(a) Chemicals



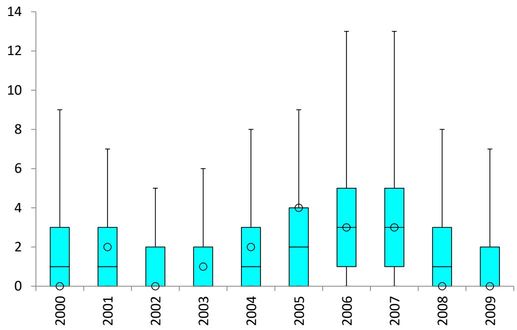
(b) Machinery



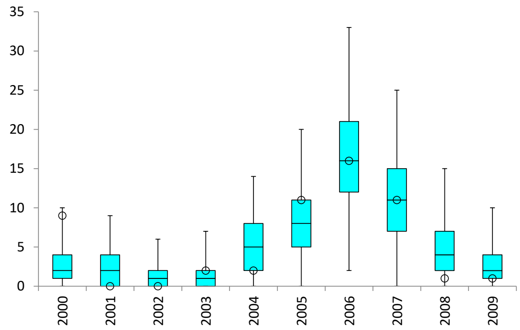
(c) Electric Appliances



(d) Transportation Equipment



(e) Retail Trade



(f) Banks

Fig. 2: The actual number of *upgrades* (circles) for *sector Chemicals, Machinery, Electric Appliances, Transportation Equipment, Retail Trade and Banks* in our sample data (from January 1st, 2000 to December 30th, 2009) as well as the boxplot per sector of the distribution of the number of upgrades during some period obtained via the simulation. As for the boxplot, the top and the bottom indicate the maximum and minimum respectively, the upper side and the lower side of each box indicate the 90th and the 10th percentile respectively, and the band inside does the median.

From these figures, it follows the annual actual rating changes in most sectors are located in the range between the 90th and the 10th percentile of the estimated distribution of the annual event number obtained by simulation on the exponential regression CQVF model.

Fig. 4 display boxplots per sector of the estimated distributions of downgrades and upgrades for both the exponential regression CQVF model and the constant CQVF model as well as the actual number of the corresponding events.

Like the results for the training data, we can see the actual downgrades are located in the range between the 90th and the 10th percentile of the simulated distribution obtained from both models in most sectors[*6]. We remark, however, that the downgrades in "Banks (sec. 28)" are far out of the range of 10% and 90% .

In some cases, we notice that the actual events are out of the range of 10% and 90% for the estimated constant CQVF model while they are in the same range for the exponential regression CQVF model. Fig. 4 shows the results for other sectors with observations of such cases. Such observations indicate that the dynamics of CQVF can be explained to some extent by employing alpha and beta of TOPIX sector indices, and that the exponential regression CQVF model fits the data a little better than the constant model. However it is not clear that the exponential regression CQVF model is significantly superior to the constant model.

4 Concluding remarks

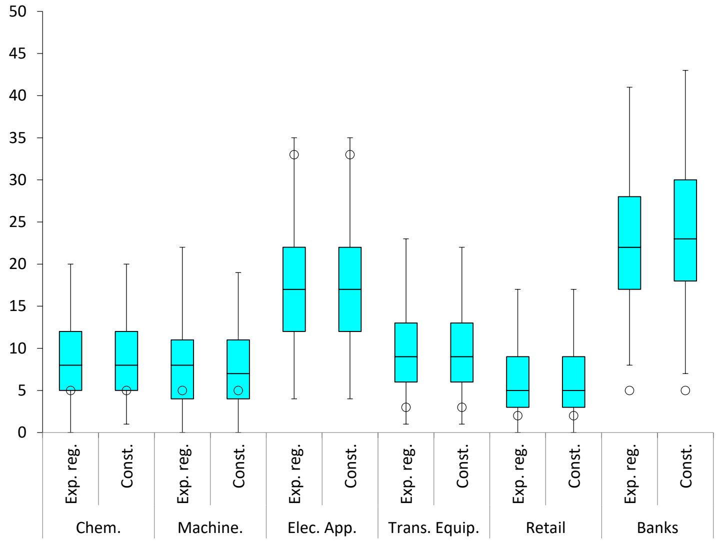
In this paper, we propose a new model of CQVF that can be specified in terms of exponential regression with some covariates. Then, we execute some empirical analyses on estimating our new exponential regression CQVF model for credit rating transitions classified into the industrial sector sub-portfolios with the historical data reported in Japan. As a result, we see that the "alpha" obtained from sector-based stock indices and the dummy for momentum effect are significant as explanatory variables of CQVF.

In addition, we examine how the actual event count is located in the estimated distribution obtained by simulation for our new CQVF model per industry sector. Consequently our new model shows a little better fit to the data than the constant CQVF model. Thus the result implies that our model can improve practical credit risk assessment.

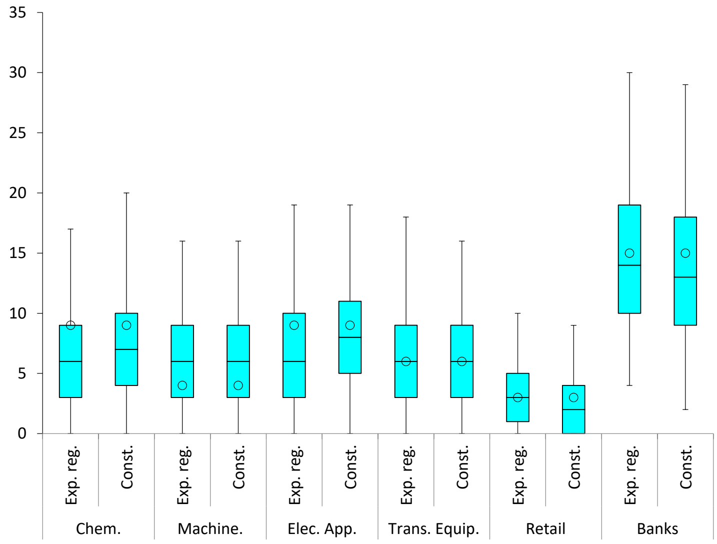
For a further improvement of CQVF model, we can consider some alternative types of our CQVF model. For example, we may also consider the model in which the coefficients differ from sector to sector unlike our presumption of common coefficients for every sector as seen in (3).

Naturally it is also a future work to find better risk factors other than the ones used in this study to improve the exponential regression CQVF model. Provided we can select some significant macro-economic variables as risk factors for CQVF, our model would be useful for stress testing of credit portfolios.

[*6] Some observations are out of the range between the 90th and the 10th percentile of the distribution obtained from the primitive model.



(a) Downgrade



(b) Upgrade

Fig. 3: The actual number of downgrades and upgrades (circles) for *sector Chemicals, Machinery, Electric Appliances, Transportation Equipment, Retail Trade and Banks* in our test data (from January 1st, 2010 to December 30th, 2013) as well as the boxplot per sector of the distribution of the number of downgrades and upgrades during some period obtained via the simulation. As for the boxplot, the top and the bottom indicate the maximum and minimum respectively, the upper side and the lower side of each box indicate the 90th and the 10th percentile respectively, and the band inside does the median.

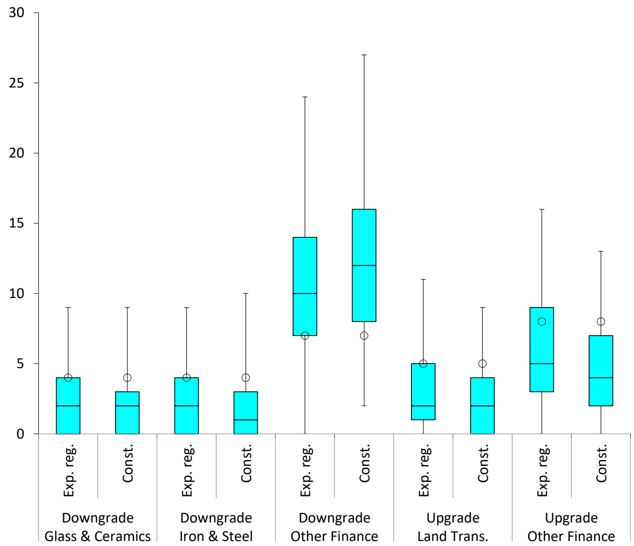


Fig. 4: The actual number of downgrades for *sector Glass & Ceramics Products, Iron & Steel and Other Financing Business*and upgrades (circles) for *sector Land Transportation and Other Financing Business*, in our test data (from January 1st, 2010 to December 30th, 2013) as well as the boxplot per sector of the distribution of the number of downgrades and upgrades during some period obtained via the simulation. As for the boxplot, the top and the bottom indicate the maximum and minimum respectively, the upper side and the lower side of each box indicate the 90th and the 10th percentile respectively, and the band inside does the median.

Appendix A Tokyo Stock Exchange 33 sector classification

Table. 2: Tokyo Stock Exchange 33 sector classification

Sec No.	Sector Name	Sec No.	Sector Name
1	Fishery, Agriculture & Forestry	18	Precision Instruments
2	Mining	19	Other Products
3	Construction	20	Electric Power& Gas
4	Foods	21	Land Transportation
5	Textiles & Apparels	22	Marine Transportation
6	Pulp & Paper	23	Air Transportation
7	Chemicals	24	Warehousing & Harbor
8	Pharmaceutical	25	Information & Communication
9	Oil & Coal Products	26	Wholesale Trade
10	Rubber Products	27	Retail Trade
11	Glass & Ceramics Products	28	Banks
12	Iron & Steel	29	Securities & Commodity Futures
13	Nonferrous Metals	30	Life Insurance & Non-life insurance
14	Metal Products	31	Other Financing Business
15	Machinery	32	Real Estate
16	Electric Appliances	33	Services
17	Transportation Equipment		

Appendix B Estimates of fixed effect

Table. 3: Estimates of fixed effect. The values in parentheses are the standard errors. The estimates which are significant under 5% significance level are marked by "$*$". Also, the actual number of downgrades or upgrades and the average number of firms in each sector are presented.

Sector No.	Down		Up		Ave of the num. of firm
	γ^i	Num. of events	γ^i	Num. of events	
1	1.523 (1.002)	1	−7.171 (219.841)	0	0.3
2	0.486 (1.001)	1	1.621 * (0.580)	3	1.8
3	0.345 (0.229)	20	0.485 (0.381)	7	26.8
4	−0.056 (0.254)	16	1.010 * (0.235)	19	32.0
5	0.412 (0.337)	9	0.493 (0.580)	3	10.2
6	0.538 (0.357)	8	1.132 * (0.411)	6	9.4
7	0.179 (0.198)	27	1.419 * (0.179)	34	46.5
8	−0.506 (0.449)	5	1.090 * (0.320)	10	15.3
9	1.018 * (0.450)	5	1.653 * (0.450)	5	3.9
10	−0.012 (0.709)	2	0.447 (1.001)	1	3.5
11	0.124 (0.502)	4	1.194 * (0.450)	5	7.4
12	0.371 (0.357)	8	2.002 * (0.263)	15	11.0
13	1.118 * (0.292)	12	1.723 * (0.357)	8	8.0
14	−0.277 (0.579)	3	0.353 (0.709)	2	7.3
15	0.499 * (0.195)	28	1.769 * (0.187)	31	34.3
16	0.734 * (0.142)	56	1.567 * (0.164)	41	56.5
17	0.635 * (0.191)	29	1.603 * (0.190)	30	32.8

Table. 4: Estimates of fixed effect. The values in parentheses are the standard errors. The estimates which are significant under 5% significance level are marked by "$*$". Also, the actual number of downgrades or upgrades and the average number of firms in each sector are presented.

Sector No.	Down		Up		Ave of the num. of firm
	γ^i	Num. of events	γ^i	Num. of events	
18	0.417	6	1.606 *	8	8.1
	(0.411)		(0.357)		
19	0.535 *	16	1.040 *	11	18.7
	(0.254)		(0.306)		
20	0.287	13	−1.698	1	18.7
	(0.281)		(1.002)		
21	0.279	20	0.648 *	11	27.8
	(0.228)		(0.306)		
22	0.098	2	1.983 *	7	4.0
	(0.709)		(0.381)		
23	2.432 *	14	1.072	2	2.6
	(0.272)		(0.709)		
24	0.182	4	−14.307	0	6.7
	(0.502)		(838.861)		
25	1.315 *	8	1.616 *	6	5.2
	(0.357)		(0.412)		
26	0.253	18	1.531 *	23	29.1
	(0.240)		(0.216)		
27	0.563 *	35	1.162 *	15	35.4
	(0.176)		(0.263)		
28	0.859 *	54	1.966 *	53	47.5
	(0.144)		(0.147)		
29	1.129 *	13	2.420 *	19	9.7
	(0.282)		(0.235)		
30	0.990 *	12	1.540 *	8	9.1
	(0.292)		(0.357)		
31	0.979 *	27	1.665 *	20	22.2
	(0.199)		(0.229)		
32	1.165 *	16	1.575 *	9	9.8
	(0.254)		(0.337)		
33	−0.455	9	1.137 *	16	28.9
	(0.336)		(0.255)		

References

[1] K. Giesecke, L. R. Goldberg and X. Ding, A top-down approach to multi-name credit, Oper. Res., **59** (2011), 283–300.

[2] K. Giesecke, B. Kim: Risk analysis of collateralized debt obligations, *Operations Research*, vol.59 (2011), pp.32–49.

[3] I. Halperin and P. Tomecek, Climbing Down from the Top: Single Name Dynamics in Credit Top Down Models, *Working paper*(2008), JP Morgan.

[4] T. Kaneko and H. Nakagawa: An application of top-down approach for credit loan portfolio based on ratings process. IMES DiscussionPaper Series 2010-J-13 (2010), Bank of Japan, (written in Japanese).

[5] M. Kunisch and M. Uhrig-Homburg: Modeling simultaneous defaults: a top-down approach, *The Journal of Fixed Income*, vol.18 (2008), pp.25–36.

[6] S. Yamanaka, H. Nakagawa and M. Sugihara: Random thinning with credit quality vulnerability factor for better risk management of credit portfolio in a top-down framework, *Japan Journal of Industrial and Applied Mathematics*, vol.33(2016), pp.321–341.

[7] S. Yamanaka, H. Nakagawa and M. Sugihara: A random thinning model with a latent factors for improvement of top-down credit risk assessment, *JSIAM Letters*, vol. 8(2016), pp.37–40.

[8] S. Yamanaka, M. Sugihara and H. Nakagawa: Modeling of Contagious Credit Events and Risk Analysis of Credit Portfolios, *Asia-Pacific Financial Markets*, vol.19 (2012), pp. 43–62.