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VAR-Based Turbulence Index: Change Point Detection for Upside Investments

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VAR-Based Turbulence Index: Change Point Detection for Upside Investments

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Abstract

Kritzman and Li (2010) introduced the turbulence index (TI) based on the Mahalanobis distance for capturing the degree of multivariate asset price "unusualness" over time. We consider adding a sign to the unusualness indicator to detect change points toward both bull and bear markets. We find that unusualness is driven by government bonds when TI is extended to vector autoregression-based TI (VTI) to eliminate autocorrelation structure and introduce signed VTI (SVTI) based on this finding. Our simulation with simple dynamic asset allocation strategies using the TI, SVTI, and a static counterpart suggests that SVTI could enhance performance compared with the other portfolios.

Keywords: Regime shifts, Change point detection, Turbulence index, Mahalanobis distance, Investment risk management, Asset allocation

1. Introduction

Dynamic asset allocation (DAA) is one of the most important processes for investors to improve their risk-adjusted returns, and it is essential for effective DAA to detect change points toward both the bear and bull markets. Researchers and practitioners have developed and applied many methods.

The hidden Markov (HM) model as developed by Baum and Petrie (1966) is one of the most popular models for such purposes and is called the regime switching model in financial literature. The most common HM model in financial literature is the HM autoregressive model, which is enabled for switching among several autoregressive (AR) models with different parameters (Hamilton 1989, 1994). To fit this kind of HM model, several regimes in a data window must be explicitly countable. However, many types of markets exist, and specifying a number of regimes through statistical data analysis is difficult. In addition, when the obtained number of regimes is large, fitting the HM model to the data becomes quite challenging.

GARCH family models and the AR model with time-varying coefficients have been also applied to represent a time series with regime shifts. This is because their time-varying parameters including volatility can absorb the differences among models in all the regimes. Thus, a number of regimes is not required to be specified. However, unlike the HM model, these models are not sensitive enough to detect change points of regimes as their time-varying parameters do not change significantly and quickly.

Meanwhile, Kritzman and Li (2010) introduced the turbulence index, which is based on the Mahalanobis distance. This measure of financial turbulence does not require information on the number of regimes, and the authors demonstrated that spikes in the index coincide with events known to be associated with financial turbulence. They also suggested several methods for applying the TI. Kritzman and Li added that "Portfolio managers can use it to stress-test portfolios by estimating VaR from the covariances that prevailed during the turbulent subsample. They can also construct portfolios that are relatively resistant to turbulence by conditioning inputs to the portfolio construction on the performance of assets during periods of turbulence. Finally, they can enhance the performance of certain risky strategies by using turbulence as a filter for scaling exposure to risk" (2010). Kritzman et al. (2012) showed regime-switching asset allocation significantly improves performance compared with an unconditional static alternative. Exposures were reduced to equities when a Markov-switching model applied to the TI indicates turbulence. Kinlaw and Turkington (2014) extended the TI by disentangling the volatility and correlation components of turbulence to derive a measure of correlation surprise and suggested that portfolio managers may be able to enhance their performance by de-risking when they observe correlation surprises coupled with heightened volatility.

Previous studies related to the TI, in general, utilized it to avoid large losses as average-realized returns tend to be low in turbulent periods. In contrast, as we show in the next section, sometimes a high-risk asset, represented by equity, strongly perform even when TI was at high levels, which would lead to large opportunity losses if portfolio managers kept their portfolios conservative.

In this study, we consider adding a sign to an "unusualness" indicator such as the TI to detect change points towards both bull and bear markets. First, we define degree of contribution of each asset class to the TI to find which asset class tends to cause spikes of the index. We find that any asset class did not always drive spikes, while TI recognizes autocorrelation structure among asset classes as part of unusualness by definition. Second, we introduce vector autoregression-based TI (VAR-based TI, VTI) to measure unusualness after eliminating the autocorrelation structure, and decompose it by following the same methodology applied to the TI. Furthermore, we introduce signed VTI (SVTI) based on our finding in the attribution analysis for VTI. Finally, we simulate simple dynamic asset allocation strategies using the TI, SVTI, and a static counterpart, a 60/40 equities/government bonds portfolio.

2. Degree of Contribution of Each Asset Class to the TI

Following Kritzman and Li (2010), we measure financial turbulence using the following multivariate distance:

$$TI_t = (\mathbf{y}_t - \boldsymbol{\mu})'\boldsymbol{\sigma}^{-1}(\mathbf{y}_t - \boldsymbol{\mu})$$

where

$\mathbf{y}_t$ = vector of asset returns for period t

$\boldsymbol{\mu}$ = sample average vector of historical returns

$\boldsymbol{\sigma}$ = sample covariance matrix of historical returns.

Kiritzman and Li (2010) showed that this statistical characterization of financial turbulence, the turbulence index, is highly consistent with events in financial history widely regarded as turbulent. In this study, we compute a weekly TI for multi-asset investment universe using returns for the following six indices representing developed market equities, emerging market equities, global high-yield bonds, emerging market debts, global investment grade corporate bonds, and global government bonds, which are typically held in a global multi-asset portfolio:

MSCI World (total return, USD unhedged)

MSCI Emerging Markets (total return, USD unhedged)

ICE BofAML Developed Markets High-Yield Constrained BB-B (USD hedged)

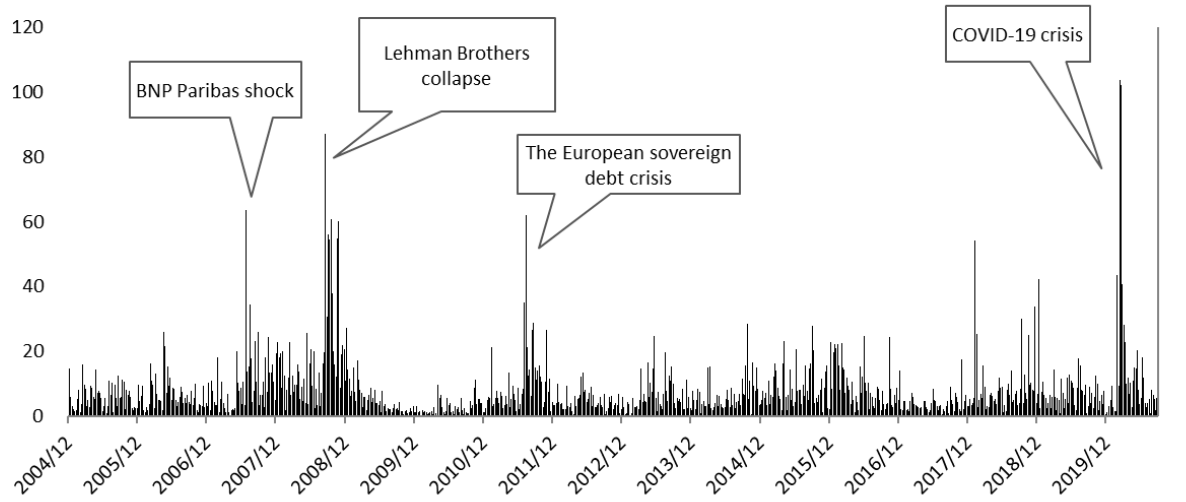
JP Morgan GBI EM Global Diversified Composite (USD unhedged)

ICE BofAML Global Broad Corporate (USD hedged)

Bloomberg Barclays Global Aggregate Government (USD hedged)

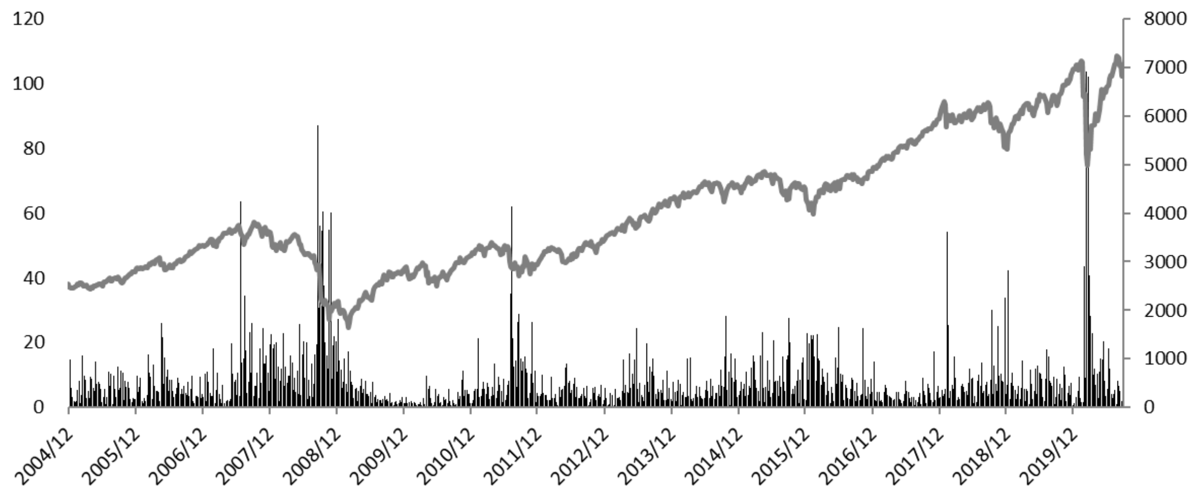
We estimate the mean and covariance using historical returns for the past 104 weeks. The TI time series begin on 6 January 2005 and end on 1 Oct 2020. **Figure 1** shows the TI and spikes in this index can be seen to coincide with events known to have been associated with financial turbulence.

Figure 1. The Turbulence Index



As we mentioned in the introduction, previous studies related to the TI generally utilized the TI to avoid large losses as average-realized returns tend to be low in turbulent periods. **Figure 2** displays historical chart of MSCI World and the TI. The TI spiked simultaneously with large sell-offs and stays at high levels for some periods thereafter, which suggests that TI could be applied as a sell signal if the market continued to fall. In contrast, if the market recovered rapidly, using the sell signal based on the TI could lead to unnecessarily conservative positions that would lose upside opportunities. For example, the TI spiked in late March 2020 which captured the COVID-19 crisis, while it remained high even after the equity market started a strong rebound.

Figure 2. TI and MSCI World (rhs)



We consider adding a sign to the TI to detect change points towards both bull and bear markets. To understand which asset class usually drives spikes in the TI, we define the degree of contribution of an asset class i to the TI for period t ($C_{i,t}$) as follows:

$$C_{i,t} = \frac{1}{TI_t}(y_{i,t} - \mu_i)^2 \sum_{j=1}^{n} \sigma^{-1}{}_{i,j}$$

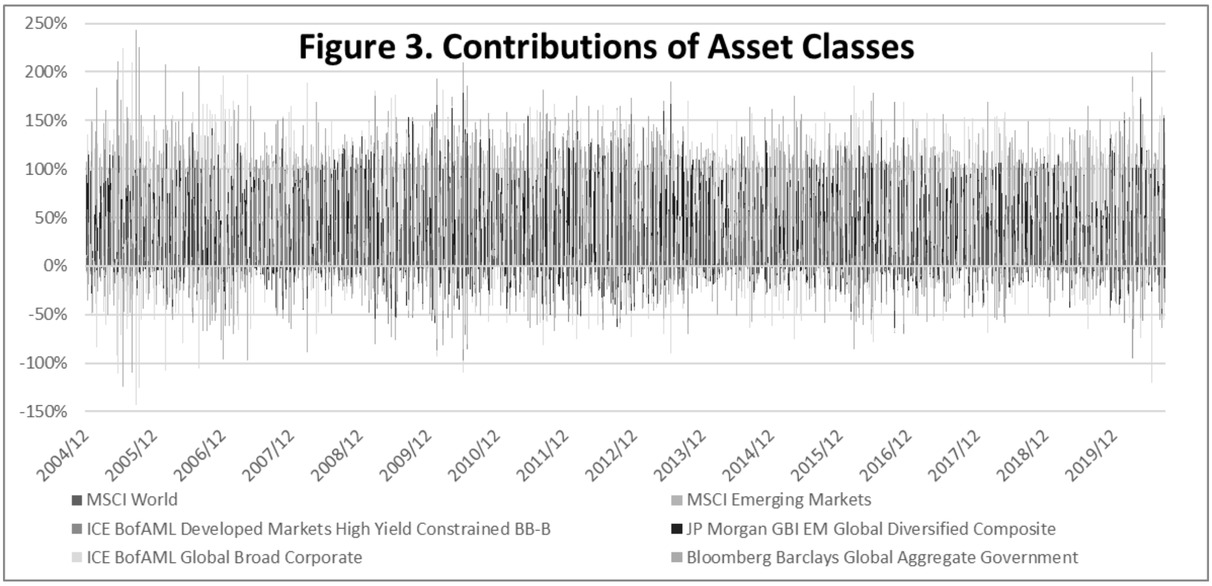
where

$$i, j = 1, 2, \cdots, n$$
$$y_{i,t} = \text{retun of asset } i \text{ for period } t$$
$$\mu_i = \text{sample average of historical returns of asset } i$$
$$\sigma^{-1}{}_{i,j} = (i, j) \text{ element of } \boldsymbol{\sigma}^{-1}$$

while

$$\sum_{i=1}^{n} C_{i,t} = \frac{1}{TI_t} \sum_{i=1}^{n} \left((y_{i,t} - \mu_i)^2 \sum_{j=1}^{n} \sigma^{-1}{}_{i,j} \right) = 1.$$

Figure 3 shows historical contributions of the asset classes. Almost no common observation can be found when the TI spikes, leaving almost no hint to add a sign.



3. VAR-Based Turbulence Index

The TI measures market unusualness and is based on historical mean and covariance, which means time-based autocorrelation structure is recognized as part of the unusualness by definition. In contrast, assets tend to maintain recent price trends in the future, and the socalled momentum strategy takes advantage of this phenomenon. The TI could be adjusted to capture unusualness after removing the autocorrelation structure and replacing the mean and the variance with the prediction error and the autocovariance of error terms, respectively. We introduce the VAR-based turbulence index (VTI) as follows:

$$VTI_t = (\mathbf{y}_t - \hat{\boldsymbol{\mu}})' \hat{\boldsymbol{\sigma}}^{-1} (\mathbf{y}_t - \hat{\boldsymbol{\mu}})$$

where

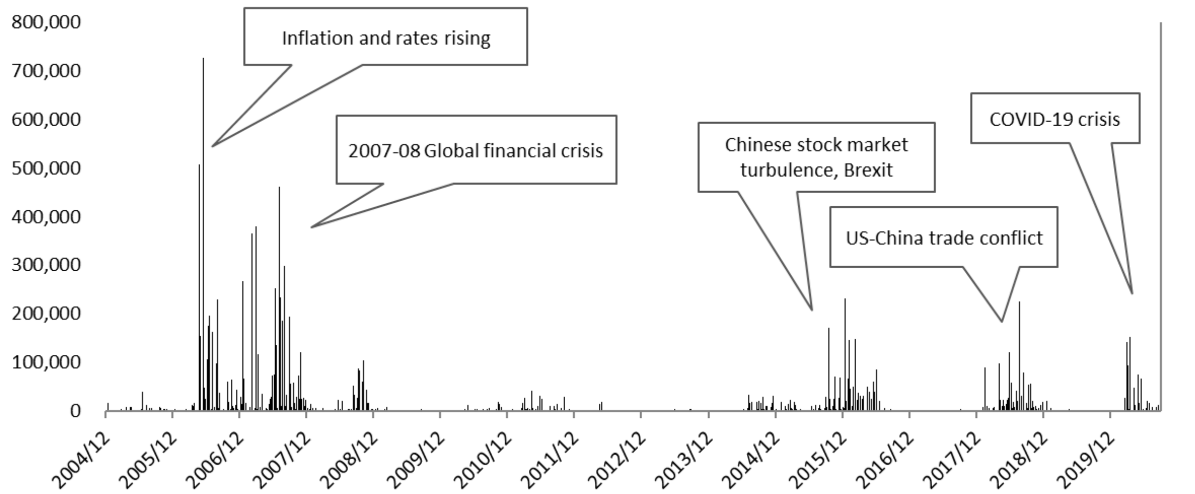
$\mathbf{y}_t$ = vector of asset returns for period t

$\hat{\boldsymbol{\mu}}$ = predicted vector of $\mathbf{y}_t$ based on VAR model

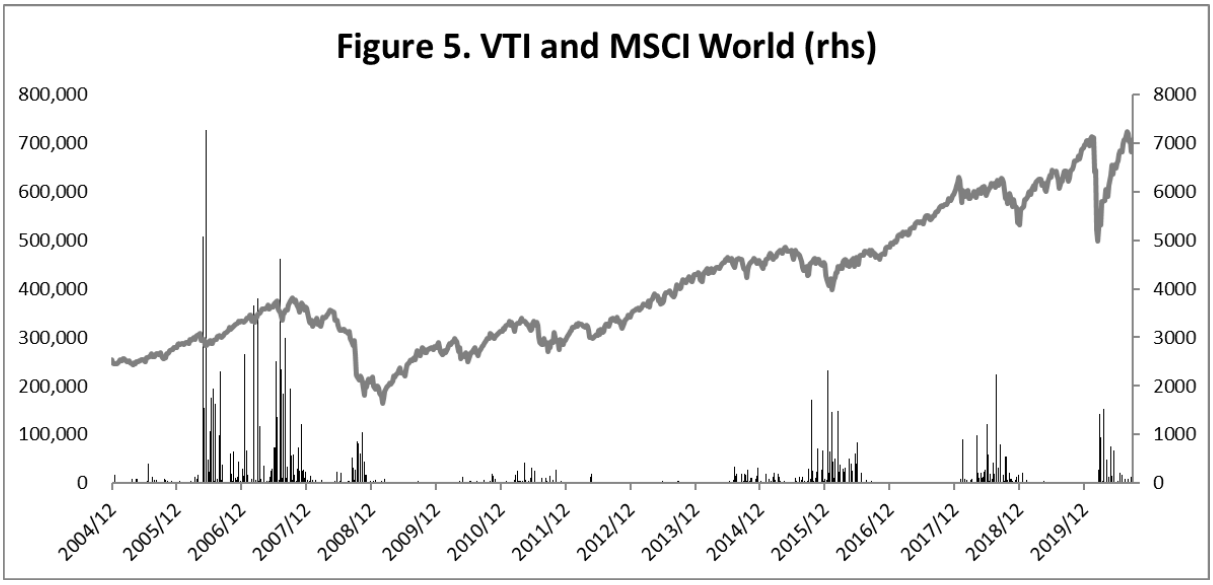
$\hat{\boldsymbol{\sigma}}$ = autocovariance matrix of error terms estimated by VAR model.

In this study, we assume a VAR model of one order and call the difference between y_t and $\hat{\mu}$ "prediction error." **Figure 4** shows the VTI calculated by the same data set as the TI. To reiterate, spikes in the VTI coincide with events known to have been associated with financial turbulence, although the events are not exactly the same as those seen in the TI spikes. Moreover, as implied in the vertical scales in Figures 1 and 4, the difference in height in the VTI is higher than that of the TI, which could make the former a clearer signal with some adjustment.

Figure 4. The VAR-based Turbulence Index



In contrast, as in the TI, applying VTI to having a sell signal could lead to unnecessarily conservative positions that would lose upside opportunities as the equity market sometimes initiated a rebound while the VTI remained at high levels, which occurred in the COVID-19 crisis period as shown in **Figure 5**.



We consider adding a sign to the VTI to detect change points towards both bull and bear markets as we tried for the TI. To understand which asset class usually drives spikes in the VTI, we define degree of contribution of each asset class to the VTI as follows:

$$\hat{C}_{i,t} = \frac{1}{VTI_t}(y_{i,t} - \hat{\mu}_i)^2 \sum_{j=1}^{n} \hat{\sigma}_{i,j}^{-1}$$

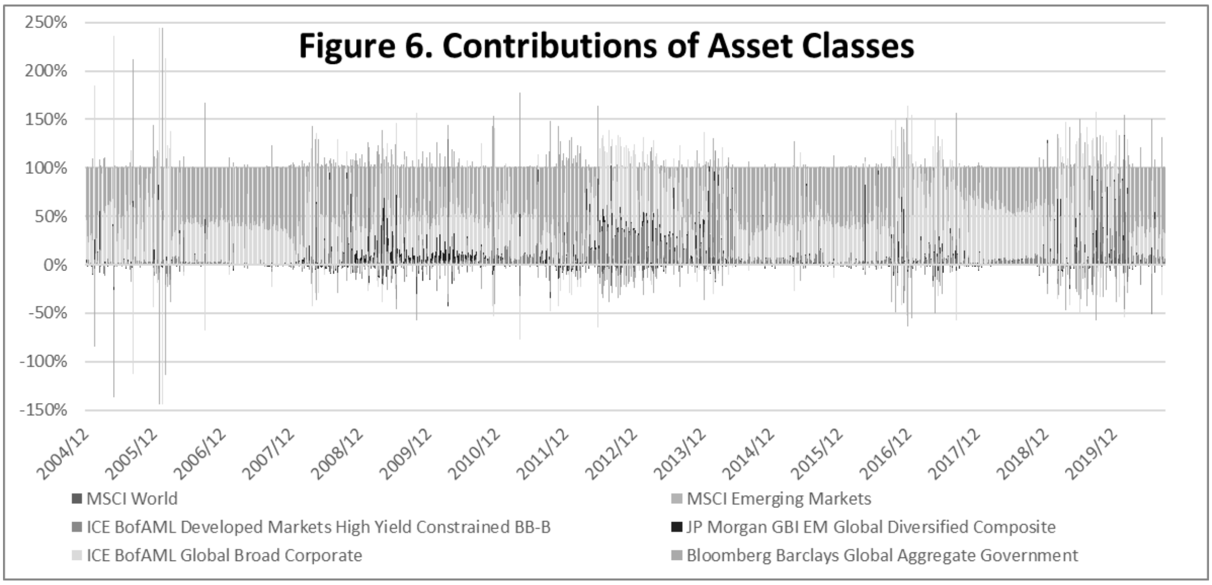
where

$$i, j = 1, 2, \cdots, n$$
$$y_{i,t} = \text{retun of asset } i \text{ for period } t$$
$$\hat{\mu}_i = \text{predicted value of } y_{i,t} \text{ based on VAR model}$$
$$\hat{\sigma}_{i,j}^{-1} = (i, j) \text{ element of } \hat{\boldsymbol{\sigma}}^{-1}$$

while

$$\sum_{i=1}^{n} \hat{C}_{i,t} = \frac{1}{VTI_t} \sum_{i=1}^{n} \left((y_{i,t} - \hat{\mu}_i)^2 \sum_{j=1}^{n} \hat{\sigma}_{i,j}^{-1} \right) = 1.$$

Figure 6 shows the historical contributions of the asset classes. In contrast to the TI, the historical attribution analysis shows which asset classes tend to determine the VTI—Global Investment Grade Corporate Bonds and Global Government Bonds. This is observed especially when the VTI spikes, probably because the impacts of the prediction errors of the two defensive asset classes are larger than the others, given that the errors and those variances are generally lower in normal market conditions.

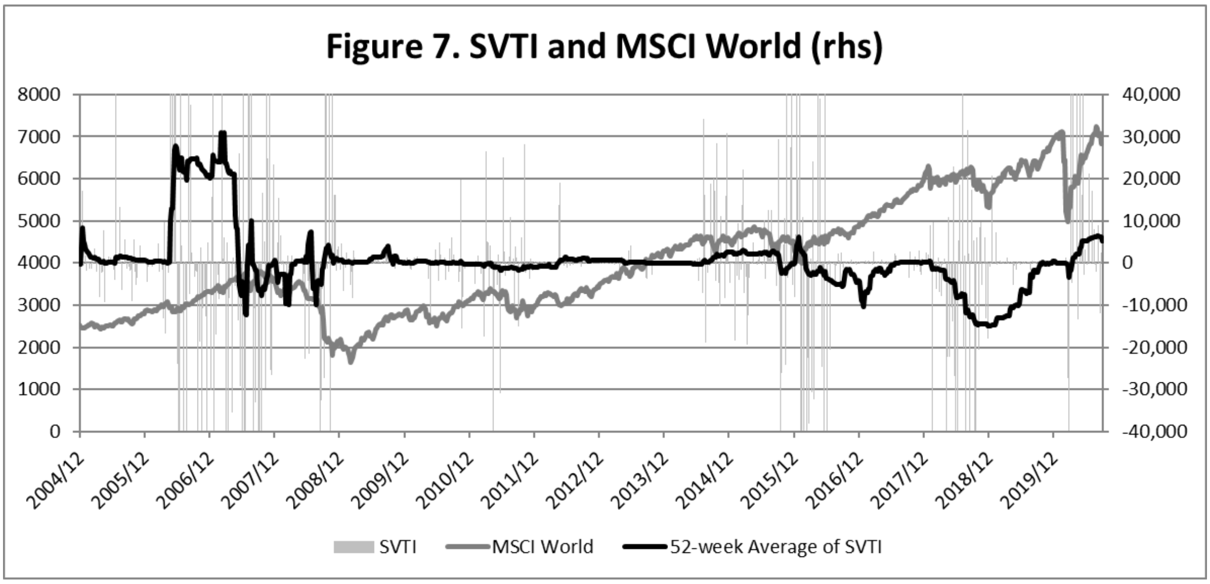


4. Signed VAR-Based Turbulence Index

As found in the previous section, two defensive fixed income assets, global investment grade corporate bonds and global government bonds, are main drivers of the VTI. In this study, we use sign of prediction error of global government bonds as sign of the following signed VAR-based turbulence index (SVTI) as interest rate is a major common factor for the two asset classes.

$$SVTI_t = VTI_t \times \text{sign of estimation error for Global Government Bonds}$$

A positive (negative) SVTI value means a realized return of global government bonds is higher (lower) than predicted based on relationship with the other asset class returns, suggesting that global government bonds is overvalued (undervalued). **Figure 7** shows the SVTI with 52-week average and MSCI world. The 52-week average is added as the index increases and decreases with exceptionally high absolute levels especially when the market is in turbulence, making it difficult to determine whether the index suggests a bull or bear market. In some periods, the trend of the SVTI leads or matches market direction, implying the effectiveness of the index as a buy/sell signal. For example, the trend of the index becomes negative before the market drop in late 2018, while it becomes positive right after bottom of COVID-19 crisis.



5. Empirical Analysis: Dynamic Asset Allocation

To test SVTI as a buy/sell signal, we design the following simple experiment. Each week, we calculate weights of developed market equities (w_E) and global government bonds (1 - w_E) in the following two dynamic asset allocation portfolios as follows:

TI Portfolio:

$$w_E = 80\% - 40\% \times \text{percentile rank of N-week average of the TI in the most recent 104 weeks}$$

SVTI Portfolio:

$$w_E = 40\% + 40\% \times \text{percentile rank of N-week average of the VTI in the most recent 104 weeks}$$

Therefore, the weight of the DME (GGB) could change from 40% to 80% (from 20% to 60%). We attempt using 13, 26, and 52 for N. Benchmark portfolio is a 60%/40% DME/GGB portfolio. Data frequency is weekly, beginning and ending on 28 Dec 2006 and 1 October 2020, respectively. Table 1 compares portfolio performances before costs. Concerning return, all the dynamic asset allocation portfolios outperform the benchmark, and the best performer is the SVTI portfolio (N = 52). The volatilities of the TI portfolios are lower than the benchmark by more than 1.00%, while those of the SVTI portfolios are higher than the benchmark; however, the degree is limited. Moreover, the TI portfolios show superior results in downside risk management to the other portfolios. In contrast, tracking errors of the SVTI portfolios are lower than the TI portfolios in general, which leads to the highest information ratio combined with the best absolute and excess return when N is 52. Essentially, dynamic tilts based on the SVTI worked most effectively when N is 52. Therefore, especially for investors who expect dynamic asset allocation to enhance return, the SVTI would be an effective buy/sell signal.

Table 1. Benchmark and Dynamic Asset Allocation Performance

	BM	TI Portfolios			SVTI Portfolios		
N	-	13	26	52	13	26	52
Return p.a.	5.55%	6.04%	5.87%	5.61%	6.00%	5.82%	6.21%
Volatility p.a.	10.77%	9.52%	9.46%	9.46%	11.12%	10.84%	10.94%
Return/volatility	0.52	0.63	0.62	0.59	0.54	0.54	0.57
Excess return	-	0.49%	0.32%	0.06%	0.45%	0.27%	0.66%
Tracking error p.a.	-	2.41%	2.24%	1.77%	1.99%	1.78%	1.77%
Information ratio	-	0.20	0.14	0.03	0.23	0.15	0.37
Max drawdown	-35.92%	-28.28%	-27.74%	-29.86%	-35.31%	-36.13%	-37.45%

6. Conclusion

For investors making dynamic asset allocations, detecting change points toward both bear and bull market is important. We introduce a decomposition methodology for the turbulence index that Kritzman and Li (2010) introduced to see contributions from asset classes. Based on this attribution analysis, we describe how to extend the TI to develop signs for predicting market direction. Finally, we present evidence showing that the extended TI, signed VAR-based turbulence index, could improve performance compared with a static alternative. Portfolio managers may be able to enhance their performance by risking / de-risking when they observe positive / negative spikes in the signed VAR-based TI, while the TI does not directly give you a signal for market direction. Future researches would study how to improve predictability of the signed VAR-based TI, or would be about a more sophisticated method to determine weights based on the index than the simple rule described in the empirical analysis.

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