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In general, Counterparty Credit Risk (CCR) in Over-the-Counter (OTC) derivatives has been significantly mitigated by margin regulations. Specifically, Initial Margin (IM) is expected to cover the increment of the exposure during Margin Period of Risk (MPoR) from a counterparty default to the actual liquidation. Since this incremental exposure is treated as a random variable at the time of the last margin delivery, IM is calculated in practice using a unified simplified calculation method called ISDA Standard Initial Margin Model (ISDA SIMM).

However, it is generally known that a default of a large financial institution, which is defined as a "Systemically Important Counterparty (SIC)" by Pykhtin and Sokol (2013), impacts financial markets, thereby increasing CCR exposure. ISDA SIMM attempts to incorporate such events in simplified conservative methods, nevertheless it does not accurately model the systemic impact.

In this study, we derive analytical approximate formulas for key CCR indicators that directly incorporate the impact of a SIC default as a discrete jump, thereby avoiding the need for computationally intensive stochastic simulations. Furthermore, we conduct numerical experiments across several jump scenarios, using a European swaption under the SABR model as a case study.

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The results suggest that, depending on the transaction type and the nature of the jump, IM may not satisfy the regulatory requirements under the impact. Consequently, our findings imply that it is crucial for measurement and management of CCR to accurately model the impact of a SIC default.

1 Introduction

Counterparty Credit Risk (CCR) in Over-the-Counter (OTC) derivatives is the credit risk that arises when a counterparty defaults while a derivative value is positive the value cannot be realized.

The mandate of the central clearing and the margin regulations has significantly reduced CCR exposure globally after the financial crisis (2008). The central clearing reduced the amount of OTC derivatives and margin regulations mitigate exposure in OTC derivatives by requiring Variation Margin (VM) and Initial Margin (IM).

In margin regulations, the CCR exposure is the value of derivatives at the time of liquidation. VM is required to cover the exposure at the default time and IM is required to cover the increment of the value during the Margin Period of Risk (MPoR), from default to liquidation. Since margins are delivered only until the default, at the default time the increment is not predictable and must be treated as a random variable. For this reason, we need to calculate IM as a stochastic problem at the last date of margin delivery.

To calculate the IM amount, we need a model for the value process and the estimation of its parameters. Although the regulations require IM to cover 99% of this increment, they do not prescribe a specific calculation method. Therefore, the International Swaps and Derivatives Association (ISDA) developed ISDA Standard Initial Margin Model (ISDA SIMM) as a unified simplified calculation model.

The regulations were adopted in 2016. However, Bank of England (2023) has recommended further refinement of MPoR risk management due to the large losses associated with the default of Archegos Capital Management (ACM, 2021). [1] In response, International Swaps and Derivatives Association (ISDA) (2023) has increased the frequency of updating the coefficients used in ISDA SIMM.

In such a discussion, Kitani and Nakagawa (2024) mentions that IM calculated by ISDA SIMM covers 99% of the increment in normal conditions, when financial markets are not stressed, but it may not satisfy the regulatory requirements when volatility rises.

In addition, it is generally known that the default of large financial institutions impacts the financial markets. Brunnermeier and Pedersen (2009) shows that the interaction between funding liquidity and market liquidity can generate self-reinforcing liquidity spirals, whereby an adverse shock to a large and leveraged intermediary can lead to abrupt and widespread price dislocations and volatility surges. Similarly, Shleifer and Vishny (2011) argues that fire sales can transform localized credit distress into systemic events by forcing asset liquidations that depress prices across the broader financial system.

[1] Bartholomew (2022) mentions that, since ACM, which was not subject to the regulations, did not deliver IM calculated by ISDA SIMM, the recommendation is not on point. However, she also notes that if ACM had delivered IM calculated by ISDA SIMM the exposure would not have been covered by IM.

Taking the default of Lehman Brothers (LB, 2008) as an example Pykhtin and Sokol (2013) defines a "Systemically Important Counterparty (SIC)" as a counterparty whose default would impact financial markets and points out that this impact may increase the CCR exposure.

Additionally, we can observe the actual default cases after LB. For instance, the default of small financial institutions like Silicon Valley Bank (SVB, 2023) and Signature Bank (SB, 2023) impacted financial markets. Conversely, the SIC default, such as ACM (2021), did not significantly impact the financial markets as the participants remained calm.

These examples show that it is difficult to predict the impact before the credit event and the impact is stochastic.

ISDA SIMM indirectly attempts to incorporate the impact by the conservative calculation method called "1+3 standard", nevertheless it does not model the uncertainty of the impact.

This study examines whether IM calculated by ISDA SIMM adequately covers the CCR exposure or not under the impact of the counterparty default.

First, we review some actual default cases of financial institutions. Next, we derive analytical approximate formulas for key CCR indicators that directly incorporate the impact of a SIC default as a discrete jump, thereby avoiding the need for computationally intensive stochastic simulations. Furthermore, we conduct numerical experiments across several jump scenarios using a European swaption under the SABR model as an example.

The results suggest that, depending on the transaction type and the nature of the jump, IM may not satisfy the regulatory requirements under the impact. Consequently, our findings imply that it is crucial for measurement and management of CCR to accurately model the impact of a SIC default.

2 Modeling the Impact on Financial Markets Due to the Default of a Counterparty

In this section, we review some actual SIC default cases in recent years and its impact on financial markets. Then, we define a mathematical model for the general CCR exposure and introduce the impact as a jump in risk factors.

2.1 Default of SICs and the Impact on Financial Markets

In 2008, following the defaults of several SICs, Summit of Financial Market and the World Economy (G20) (2009) decided to implement several regulations to reduce the CCR. Then, central clearing of standard derivatives and the delivery of VM and IM in non-cleared derivatives were mandated.

In particular, the default of LB significantly impacted financial markets, and it has become clear that transactions with SICs involve not only individual credit risk but also systemic risk.

[Pykhtin and Sokol (2013)] points out that this impact may increase the CCR exposure through the jump in risk factors of derivatives. [2]

In fact, the default of Long Term Capital Management (LTCM, 1998) and LB impacted financial markets, as shown in Fig. 1, with a decline in the interest rate and a spike in their implied volatility.

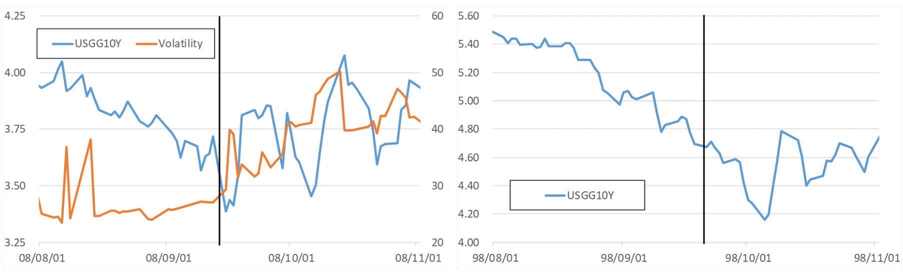

Fig. 1: Financial markets at the default time of LB (Left) and LTCM (Right). The vertical black line indicates the default date.

However, in recent years, we can observe the similar default cases.

For instance, the default of ACM, which is a large fund and according to Bouveret and Haferkor (2022) its default inflicted substantial losses upon many G-SIBs, did not impact the financial markets, as shown in Fig. 2. On the other hand, the default of SVB and SB, which are non-G-SIBs and only regional banks in the U.S., impacted the financial markets with a decline in the interest rate and a spike in their implied volatility.

These examples demonstrate that the impact is stochastic and motivate our modeling of SIC default as a stochastic jump in risk factors.

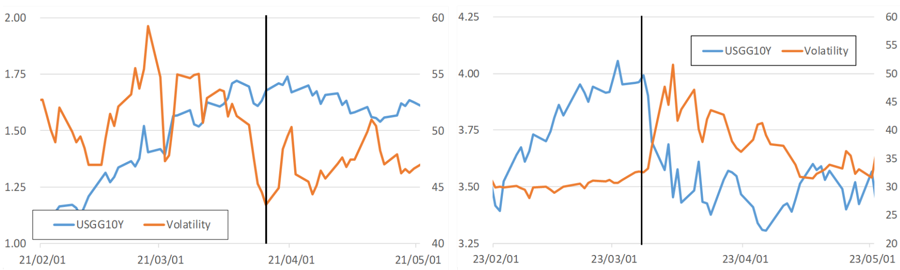

Fig. 2: Financial markets at the default time of ACM (Left) and SVB/SB (Right). The vertical black line indicates the default date.

In the next section, we define a general mathematical model and the CCR exposure.

[2] Pykhtin and Sokol (2013) introduces this concept as being slightly broader than G-SIBs.

2.2 Counterparty Risk and Initial Margin

In this study, we introduce a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ assuming that the probability measure $\mathbf{P}$ is the physical measure and denote the value process of derivatives with maturity T in the continuous period $[0, T]$ as V and the SIC default time as the stopping time τ. Furthermore, we assume that V can be calculated using the underlying asset price X and its volatility σ, and that the filtration $(\mathcal{F}_t)_{t\in[0,T]}$ provides information generated by all random variables excluding the impact of the counterparty default.

$$V(t) = V(t, X(t), \sigma(t))$$

In general, V should incorporate CCR. However, in this study, since V is used as the CCR exposure at the SIC default time, we model V excluding credit risk such as Credit Valuation Adjustment (CVA).

In derivatives, CCR arises as the credit risk that if counterparty defaults while the value of the derivatives is positive ($V > 0$), we cannot receive the value.

Here, if the counterparty defaults at time t, the value of the derivative continues to change stochastically until the liquidation at time $t + \delta^*$. We denote this period, MPoR, as δ^*. To mitigate this risk, regulations require OTC derivatives to deliver two types of margin, VM and IM.

In this study, if the counterparty defaults at time t, the margin that should be calculated and delivered at time t is treated as being processed at time $t-$.

Therefore, the CCR exposure is expressed as the Positive Exposure (PE) defined as the value after liquidation minus the pre-default value and IM.

In this study since we calculate the CCR at default time, PE is calculated under the condition of $\tau = t$.

$$\mathrm{PE}(t) = \left(V(\tau + \delta^*) - \mathrm{VM}(\tau-) - \mathrm{IM}(\tau-)\right)^+ \Big|_{\tau=t} . \tag{1}$$

For the sake of simplicity, unless otherwise specified, the following discussion will be based on the condition $\tau = t$, and such specifications will be omitted.

VM directly covers the value V at the counterparty default time, and can be expressed as,

$$\mathrm{VM}(t-) = V(t-) = \lim_{s\to t} V(s).$$

After the default, the value V changes stochastically until the liquidation. Although we need to cover $V(t + \delta^*)$, VM is delivered until the time t. To cover the increment of the value V from default to liquidation, the regulations require IM. IM covers the increment of the exposure in MPoR.

$$\begin{aligned} V(t + \delta^*) - \mathrm{VM}(t-) &= V(t + \delta^*) - V(t-) \\ &= V(t + \delta^*) - V(t) + (V(t) - V(t-)) , \end{aligned}$$

However, we need to calculate and deliver IM at time $t-$, so, $V(t + \delta^*)$ is not predictable at the default time $t-$. The regulations require IM to cover 99% of this stochastic increment.

$$\mathrm{IM}^{Reg}(t-) = \mathrm{ess.inf}\left\{ y \in \mathbb{R} \;\middle|\; \mathbf{P}\left(V(\tau+\delta^*) - V(\tau-) \leq y \;\middle|\; \mathcal{F}_\tau, \tau = t \right) \leq 1\% \right\}.$$

To calculate IM amount as required by regulations, we need the model of value processes and the estimation of their parameters. However, since they depend on the academic research progress and the management policies of financial institutions, regulations avoid specifying a concrete calculation method. Therefore, International Swaps and Derivatives Association (ISDA) (2024) developed ISDA SIMM as a unified, simplified calculation method.

In this method, IM is calculated using the sensitivity calculated by each financial institution and the SIMM coefficients (RW and VRW, corresponding to the volatility of each risk factor) estimated from historical data published by ISDA. The outline of the calculation formula is as follows. [3]

$$\mathrm{IM}(t) = \mathrm{IM}_{\mathrm{Delta}}(t) + \mathrm{IM}_{\mathrm{Vega}}(t) + \mathrm{IM}_{\mathrm{Curvature}}(t),$$

where,

$$\begin{aligned}
\mathrm{IM}_{\mathrm{Delta}}(t) &= \mathrm{RW} \cdot V_x(t, X(t), \sigma(t)) \cdot 1\mathrm{bp},\\
\mathrm{IM}_{\mathrm{Vega}}(t) &= \mathrm{VRW} \cdot \sigma(t) \cdot V_\sigma(t, X(t), \sigma(t)) \cdot 1\mathrm{bp},\\
\mathrm{IM}_{\mathrm{Curvature}}(t) &= \min\left\{1, \frac{14\mathrm{Days}}{\text{Time to maturity}}\right\} \cdot \sigma(t) \cdot V_\sigma(t, X(t), \sigma(t)) \cdot 1\mathrm{bp}.
\end{aligned}$$

Since ISDA SIMM was developed in response to the financial crisis, ISDA SIMM attempts to incorporate the impact indirectly. ISDA SIMM adopts the conservative method, called "1+3 standard", to estimate SIMM coefficients corresponding to the volatility of the risk factors. These coefficients are estimated by historical data that consists of one stress year [4] plus three non-stress years. Thus, IM is conservatively calibrated upward assuming a certain jump. On the other hand, it essentially aggregates the uncertain impact, which is 25 % of jump and 75 % of no jump, into a single coefficient (RW for underlying asset and VRW for its volatility). In this sense, it does not accurately model the uncertainty of the impact. [5]

2.3 Impact on financial markets due to the SIC default

In this section, we model the financial market and the impact of a certain SIC default.

[3] We focus on the general CCR. Therefore, we disregard IM related to Base Correlation specific to credit risk and related to concentration risk.

[4] The data is selected to maximize volatility among all historical data. At the time of writing, data including the default of LB may be used.

[5] In addition, ISDA SIMM requires financial institutions to conduct "1+3 back test". This involves conducting a back test using historical data on the current position. Here too, historical data that includes the most recent three years plus one year of shock periods[4] is used to intentionally increase volatility.

First, we assume that the processes X and σ for $t \neq \tau$ are governed by a generalized stochastic volatility model.

$$\begin{aligned} dX(t) &= \mu_X(t, X(t), \sigma(t))dt + \sigma_X(t, X(t), \sigma(t))dW_X(t), \\ d\sigma(t) &= \mu_\sigma(t, X(t), \sigma(t))dt + \sigma_\sigma(t, X(t), \sigma(t))dW_\sigma(t). \end{aligned}$$

Here, $\mu_X, \mu_\sigma, \sigma_X, \sigma_\sigma$ are the drifts and the volatilities for the underlying asset and its volatility, respectively, and W_X, W_σ are assumed to be standard Brownian motions under a physical probability measure $\mathbf{P}$ with correlation $\rho \in [-1, 1]$.

In this study, similar to the prior research and ISDA SIMM, we introduce the impact of the SIC default as a jump in the risk factors that constitute the value of derivative price. We denote the jump in X and σ as a random variable $J = (J_X, J_\sigma)$. We assume that the filtration $\mathcal{F}$ and J are independent.

$$X(\tau) = X(\tau-) + J_X, \quad \sigma(\tau) = \sigma(\tau-) + J_\sigma.$$

Then, we can decompose the equation (1) into the jump due to the default and the change in value following the jump.

$$\mathrm{PE}(t) = \Big((V(\tau + \delta^*) - V(\tau) - \mathrm{IM}(\tau-)) + (V(\tau) - V(\tau-)) \Big)^+ \Big|_{\tau = t}. \tag{2}$$

From Ito's formula, it follows that if the function V is a $C^{1,2,2}$-function, we can express the value process of V as,

$$\begin{aligned} dV(t) &= dV(t, X(t), \sigma(t)) \\ &= V_t(t)dt + V_x(t)dX(t) + V_\sigma(t)d\sigma(t) + \frac{1}{2}V_{xx}(t)d\langle X\rangle(t) + \frac{1}{2}V_{\sigma\sigma}(t)d\langle\sigma\rangle(t) + V_{x\sigma}(t)d\langle X\sigma\rangle(t) \\ &= V_x(t)\sigma_X(t, X(t), \sigma(t))dW_X(t) + V_\sigma(t)\sigma_\sigma(t, X(t), \sigma(t))dW_\sigma(t) \\ &\quad + \Big[V_t(t) + V_x(t)\mu_X(t, X(t), \sigma(t)) + \frac{1}{2}V_{xx}(t)\sigma_X^2(t, X(t), \sigma(t)) \\ &\quad + V_\sigma(t)\mu_\sigma(t, X(t), \sigma(t)) + \frac{1}{2}V_{\sigma\sigma}(t)\sigma_\sigma^2(t, X(t), \sigma(t)) \\ &\quad + \rho V_{x\sigma}(t)\sigma_X(t, X(t), \sigma(t))\sigma_\sigma(t, X(t), \sigma(t)) \Big] dt. \end{aligned} \tag{3}$$

Strictly speaking, the value change $V(t + \delta^*) - V(t)$ is defined by the integral $\int_t^{t+\delta^*} dV(s)$. In regulatory frameworks, Basel Committee on Banking Supervision (BCBS) (2016) sets MPoR δ^* to 10 business days and this horizon is widely adopted in risk management. [6]

[6] In the numerical experiments conducted later, we use 14 days ($\delta^* = 14\text{Days}/365\text{Days}$).

Since MPoR is sufficiently short, we can apply a discrete approximation to equation (3) using standard normal variables Z_X and Z_σ with correlation $\rho \in [-1, 1]$. [7]

$$W_X(t+\delta^*) - W_X(t) \simeq \sqrt{\delta^*} Z_X, \quad W_\sigma(t+\delta^*) - W_\sigma(t) \simeq \sqrt{\delta^*} Z_\sigma,$$

Then, the first term of equation (2) can be expressed as follows.

$$\begin{aligned}
V(t+\delta^*) - V(t) &\simeq A_D(t)Z_X + A_V(t)Z_\sigma + A_\delta(t),\\
A_D(t) &= A_D(t-, J) = V_x(t-, J)\sigma_X(t-, J)\sqrt{\delta^*},\\
A_V(t) &= A_V(t-, J) = V_\sigma(t-, J)\sigma_\sigma(t-, J)\sqrt{\delta^*},\\
A_\delta(t) &= A_\delta(t-, J)\\
&= \Big(V_t(t-, J) + V_x(t-, J)\mu_X(t-, J) + \frac{1}{2}V_{xx}(t-, J)\sigma_X^2(t-, J)\\
&\quad + V_\sigma(t-, J)\mu_\sigma(t-, J) + \frac{1}{2}V_{\sigma\sigma}(t-, J)\sigma_\sigma^2(t-, J)\\
&\quad + \rho V_{x\sigma}(t-, J)\sigma_X(t-, J)\sigma_\sigma(t-, J)\Big)\delta^*.
\end{aligned}$$

We label the jump in value due to the SIC default, the second term of equation (2), as,

$$A_J(t) = A_J(t-, J) = V(t) - V(t-) = V(t-, J) - V(t-),$$

Here, we set $A_C(t) = A_\delta(t) - \mathrm{IM}(t-)$, then equation (2) becomes,

$$\mathrm{PE}(t) = \left(A_D(t)Z_X + A_V(t)Z_\sigma + A_C(t) + A_J(t)\right)^+. \tag{4}$$

Therefore, we can see that PE is calculated by the partial derivatives of V.

In the next section, we will define CCR indicators using PE and derive their approximate formulas.

3 Analytical Approach to CCR for SICs

In this section, we introduce the general CCR indicators and derive the approximate formulas under a jump in risk factors. Furthermore, to analyze the risk in greater detail, we decompose PFE, one of the indicators, into Factor PFE of Delta, Vega, Curvature and Jump components.

3.1 CCR Indicators and Approximate Formulas

We focus on Mratio, PFE, and EPE as representative CCR indicators.

[7] To assess the magnitude of the approximation error, we compare the discrete approximation with a multi-step Monte Carlo simulation for the swaption considered in the numerical experiments conducted later in this study. The Monte Carlo simulation, based on 1,000,000 trials, partitions MPoR into 14 steps (i.e., 1 step = 1 day), and yields a mean value of 0.11243, while the discrete approximation produces 0.11171. The absolute difference (0.00072) corresponds to less than 1% of the swaption value.

Kitani and Nakagawa (2024) defines Margin Conservation Ratio (Mratio) as the probability that PE equals zero.

$$\mathrm{Mratio}(t) = \mathbf{P}\left(\mathrm{PE}(t) = 0 | \mathcal{F}_{t-}\right). \tag{5}$$

where Φ is the cumulative distribution function of the standard normal distribution, and we also define ϕ as its probability density function, which will be used in later sections.

Mratio serves to verify whether regulatory requirements are satisfied or not. Regulations require IM to cover 99 % of the loss, so if Mratio is 99 % or higher, the regulatory requirements are satisfied. Conversely, if Mratio falls below 99 %, the regulatory requirements are not satisfied. We then quantify the amount of risk.

Gregory (2020) defines Potential Future Exposure (PFE) as the 99th percentile of losses and Expected Positive Exposure (EPE) as the expectation of PE. PFE is conceptually similar to Value at Risk (VaR).

$$\begin{aligned}
\mathrm{PFE}(t) &= \operatorname{ess.inf}\left\{y \in \mathbb{R} \;\middle|\; \mathbf{P}\left(\mathrm{PE}(t) \geq y \mid \mathcal{F}_{t-}\right) \leq 0.01\right\}, \\
\mathrm{EPE}(t) &= \mathbf{E}\left[\mathrm{PE}(t) \mid \mathcal{F}_{t-}\right].
\end{aligned} \tag{6}$$

Both PFE and EPE are calculated in monetary terms. In this study, we measure the amount of the loss by PFE which is highly compatible with VaR commonly used in risk management. Although EPE is not employed in our numerical experiments, it is widely used in CVA calculations. In the numerical experiments conducted later, we first examine whether IM satisfies regulatory requirements or not by checking Mratio. If Mratio is below 99%, we check PFE to measure the amount of the loss.

In the case of No-Jump, Kitani and Nakagawa (2024) simplifies PE employing the random variable Z that follows a standard normal distribution.

$$\begin{aligned}
\mathrm{PE}(t) &\simeq A_Z(t) Z + A_M(t), \qquad (7) \\
A_Z(t) &= \sqrt{A_D^2(t) + 2\rho A_D(t) A_V(t) + A_V^2(t)}, \\
A_M(t) &= A_C(t) + A_J(t), \\
Z &= \frac{A_D(t) Z_X + A_V(t) Z_\sigma}{A_Z(t)}.
\end{aligned}$$

Then, since A_Z and A_M are $\mathcal{F}_t$-measurable functions, we can derive the approximate formulas for the CCR indicators as follows.

$$\begin{aligned}
\mathrm{Mratio}(t) &\simeq \Phi\left(-\frac{A_M(t)}{A_Z(t)} \;\middle|\; \mathcal{F}_{t-}\right), \\
\mathrm{PFE}(t) &\simeq \operatorname{ess.inf}\left\{y \in \mathbb{R} \mid \mathbf{P}\left(\left(A_Z(t) Z + A_M(t)\right)^+ \geq y \;\middle|\; \mathcal{F}_{t-}\right) \leq 0.01\right\}, \\
\mathrm{EPE}(t) &\simeq \mathbf{E}\left[\left(A_Z(t) Z + A_M(t)\right)^+ \;\middle|\; \mathcal{F}_{t-}\right].
\end{aligned}$$

Next, we consider the jump $J = (J_X, J_\sigma)$. Ideally, the jump distribution should be defined as a parametric function. However, since there are few examples of SIC defaults, it is difficult to estimate the parameters. In reality, stress scenarios under a certain assumption are more appropriate. Therefore, in this study, we assume J as a random variable with discrete probability distribution.

$$n = 1, 2 \cdots N, \quad j_n = (j_{n,X}, j_{n,\sigma}), \quad p_n = \mathbf{P}(J = j_n).$$

Here we set,

$$A_{n,Z}(t) = A_Z(t-, j_n), \quad A_{n,M}(t) = A_M(t-, j_n).$$

Then, $A_{n,Z}$ and $A_{n,M}$ are $\mathcal{F}_{t-}$-measurable functions. Since $\mathcal{F}$ and J are independent, applying Kitani and Nakagawa (2024) equations (5) and (6) are equivalent to,

$$\begin{aligned}
\text{Mratio}(t) &= \sum_{n=1}^{N} \Phi\left(-\frac{A_{n,M}(t)}{A_{n,Z}(t)}\right) p_n(t), \\
\text{EPE}(t) &= \sum_{n=1}^{N} \left[\frac{A_{n,Z}(t)}{\sqrt{2\pi}} e^{-\frac{A_{n,M}^2(t)}{2A_{n,Z}^2(t)}} + A_{n,M}(t)\Phi\left(\frac{A_{n,M}(t)}{A_{n,Z}(t)}\right)\right] p_n(t),
\end{aligned}$$

For PFE, let $\text{Mratio}'(t, y) = \mathbf{P}\left((A_Z(t)Z - A_M(t) - y)^+ = 0 \,\middle|\, \mathcal{F}_{t-}\right)$. Then,

$$\begin{aligned}
\text{PFE}(t) &= \text{ess.inf}\left\{y \in \mathbb{R} \,\middle|\, \mathbf{P}\left((A_{n,Z}(t)Z + A_M(t-))^+ \geq y \,\middle|\, \mathcal{F}_{t-}\right) \leq 0.01\right\}, \\
&= \text{ess.inf}\left\{y \in \mathbb{R} \,\middle|\, 1 - \text{Mratio}'(t, y) \leq 0.01\right\}, \\
&= \text{ess.inf}\left\{y \in \mathbb{R} \,\middle|\, \text{Mratio}'(t, y) \geq 0.99\right\}.
\end{aligned}$$

Therefore, PFE can be evaluated numerically. [8]

Thus the CCR indicators can be computed once J and the partial derivatives of V are specified.

3.2 Factor Decomposition of Exposure

Furthermore, to analyze the CCR we decompose PFE into four factors: Delta (D), Vega (V), and Curvature (C), which are the basic factors for calculating IM in ISDA SIMM, and Jump (J).

First, PE derived in equation (4) can be decomposed into four factor PEs.

$$\text{PE}(t) = (\text{PE}_D(t-) + \text{PE}_V(t-) + \text{PE}_C(t-) + \text{PE}_J(t-))^+,$$

[8] Since $A_{n,Z}(t) > 0$ from equation (7), Mratio$'$ is monotonically decreasing in y, and therefore PFE can be uniquely calculated numerically.

$$\frac{\partial \text{Mratio}'}{\partial y}(t, y) = -\sum_{n=1}^{N} \phi\left(\frac{A_{n,M}(t-) - y}{A_{n,Z}(t)}\right) \frac{p_n(t)}{A_{n,Z}(t)} \leq 0$$

where,

$$
\begin{aligned}
\mathrm{PE}_D(t) &= A_D(t)Z_X + V_x(t)\mu_X(t)\delta^* - \mathrm{IM}_{Delta}(t-),\\
\mathrm{PE}_V(t) &= A_V(t)Z_\sigma + V_\sigma(t)\mu_\sigma(t)\delta^* - \mathrm{IM}_{Vega}(t-),\\
\mathrm{PE}_C(t) &= A_C(t) - \left(V_x(t)\mu_X(t) + V_\sigma(t)\mu_\sigma(t)\right)\delta^* + \mathrm{IM}_{Delta}(t-) + \mathrm{IM}_{Vega}(t-),\\
\mathrm{PE}_J(t) &= A_J(t).
\end{aligned}
$$

Then, based on this decomposition, we introduce four factor PFEs.

For explicit calculations, similar to equation (7), we aggregate Z_X and Z_σ into Z and define PE_Z.

$$
\begin{aligned}
\mathrm{PE}_Z(t) &= \mathrm{PE}_D(t) + \mathrm{PE}_V(t)\\
&= A_Z(t)Z + \left(V_x(t)\mu_X(t) + V_\sigma(t)\mu_\sigma(t)\right)\delta^* - \left(\mathrm{IM}_{Delta}(t-) + \mathrm{IM}_{Vega}(t-)\right).
\end{aligned}
$$

Next, we define the factor PFEs with the following conditional expectations.

$$
f \in \{D, V, Z, C, J\}, \quad \mathrm{PFE}_f(t) = \mathbf{E}\left[\mathrm{PE}_f(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right].
$$

We can see that the sum of PFE_D and PFE_V equals PFE_Z under this definition.

$$
\begin{aligned}
\mathrm{PFE}_D(t) + \mathrm{PFE}_V(t) &= \mathbf{E}\left[\mathrm{PE}_D(t) + \mathrm{PE}_V(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right]\\
&= \mathbf{E}\left[\mathrm{PE}_Z(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right] = \mathrm{PFE}_Z(t).
\end{aligned}
$$

We can confirm that under this definition the aggregation of all factor PFEs reproduces the original PFE.

$$
\begin{aligned}
&\mathrm{PFE}(t) = \mathrm{PFE}_D(t) + \mathrm{PFE}_V(t) + \mathrm{PFE}_J(t) + \mathrm{PFE}_C(t),\\
&\mathrm{PFE}_Z(t) = \mathrm{PFE}_D(t) + \mathrm{PFE}_V(t).
\end{aligned}
$$

We set J as a discrete random variable with a set of N elements, $\{j_n \mid n = 1, \cdots, N\}$. Then, we define factor $\mathrm{PE}_{n,f}$ and $\mathrm{PFE}_{n,f}$ in a form further conditioned by J.

$$
\begin{aligned}
&f \in \{D, V, Z, C, J\}, \quad n = 1, \cdots, N,\\
&\mathrm{PE}_{n,f}(t) = \mathrm{PE}_f(t)\,\big|_{J=j_n} = \mathrm{PE}_f(t-, j_n),\\
&\mathrm{PFE}_{n,f}(t) = \mathbf{E}\left[\mathrm{PE}_{n,f}(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right].
\end{aligned}
$$

Since J are disjoint sets, from Bayes' formula, factor PFEs can be calculated as follows,

$$
\mathrm{PFE}_f(t) = \sum_{n=1}^{N} \mathbf{E}\left[\mathrm{PE}_{n,f}(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right] \mathbf{P}\left(J = j_n \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right)
$$

$$
\begin{aligned}
&= \sum_{n=1}^{N} \mathrm{PFE}_{n,f}(t) \frac{\mathbf{P}\left(J = j_n \cap \mathrm{PE}(t) = \mathrm{PFE}(t)\right)}{\mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t)\right)} \\
&= \frac{\sum_{n=1}^{N} \mathrm{PFE}_{n,f}(t) \cdot \mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t) \;\middle|\; J = j_n\right) p_n(t)}{\sum_{n=1}^{N} \mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t) \;\middle|\; J = j_n\right) p_n(t)}.
\end{aligned}
$$

Here, since $\mathrm{PE}_{n,C}$ and $\mathrm{PE}_{n,J}$ are functions that do not contain random variables, both conditional factor PFEs can be treated as constants.

$$
\mathrm{PFE}_{n,C}(t) = \mathrm{PE}_{n,C}(t), \quad \mathrm{PFE}_{n,J}(t) = \mathrm{PE}_{n,J}(t).
$$

Then, $\mathrm{PFE}_{n,Z}$ can be obtained as follows.

$$
\begin{aligned}
\mathrm{PFE}_{n,Z}(t) &= \mathrm{PFE}(t) - \left(\mathrm{PFE}_{n,C}(t) + \mathrm{PFE}_{n,J}(t)\right) \\
&= \mathrm{PFE}(t) - \left(\mathrm{PE}_{n,C}(t) + \mathrm{PE}_{n,J}(t)\right).
\end{aligned}
$$

Furthermore, since $\mathrm{PE}_{n,Z}$ is a random variable that follows a normal distribution, the conditional probability can be calculated as follows.

$$
\begin{aligned}
\mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t) \;\middle|\; J = j_n\right) &= \mathbf{P}\left(\mathrm{PE}_{n,Z}(t) = \mathrm{PFE}_{n,Z}(t)\right) \\
&= \mathbf{P}\left(Z = \frac{\mathrm{PFE}_{n,Z}(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]}{A_Z(t-)}\right) \\
&= \phi\left(\frac{\mathrm{PFE}_{n,Z}(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]}{A_Z(t-)}\right). \\
\mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right] &= \left(V_x(t)\mu_X(t) + V_\sigma(t)\mu_\sigma(t)\right)\delta^* - \left(\mathrm{IM}_{Delta}(t-) + \mathrm{IM}_{Vega}(t-)\right).
\end{aligned}
$$

Since PE_D and PE_V are random variables that follow a normal distribution, the conditional expectation $\mathrm{PFE}_{n,D}$ can be calculated employing their correlation ρ_Z between PE_D and PE_V as follows. [9]

$$
\begin{aligned}
\mathrm{PFE}_{n,D}(t) &= \mathbf{E}\left[\mathrm{PE}_{n,D}(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}\right] \\
&= \mathbf{E}\left[\mathrm{PE}_{n,D}(t) \;\middle|\; \mathrm{PE}_{n,Z}(t) = \mathrm{PFE}_Z(t)\right] \\
&= \mathbf{E}\left[\mathrm{PE}_{n,D}(t)\right] \\
&\quad + \frac{A_D^2(t) + \rho_Z A_D(t) A_V(t)}{A_Z^2(t)} \left(\mathrm{PFE}_Z(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]\right), \qquad (8) \\
\rho_Z &= \frac{A_Z^2(t) - A_D^2(t) - A_V^2(t)}{2A_D(t)A_V(t)}.
\end{aligned}
$$

[9] See the Appendix for a detailed derivation.

$$\mathbf{E}\left[\mathrm{PE}_{n,D}(t)\right] = V_x(t)\mu_X(t)\delta^* - \mathrm{IM}_{Delta}(t-).$$

Similarly, we can calculate PFE_V as follows.

$$\mathrm{PFE}_{n,V}(t) = \mathbf{E}\left[\mathrm{PE}_{n,V}(t)\right] + \frac{A_V^2(t) + \rho_Z A_D(t)A_V(t)}{A_Z^2(t)}\left(\mathrm{PFE}_Z(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]\right).$$

$$\mathbf{E}\left[\mathrm{PE}_{n,V}(t)\right] = V_\sigma(t)\mu_\sigma(t)\delta^* - \mathrm{IM}_{Vega}(t-).$$

4 Numerical Experiments for Swaptions under the SABR Model

In this section, we apply the CCR indicators in numerical experiments on a European swaption under the SABR model. We then conduct numerical experiments across several jump scenarios. As a result, we can confirm that IM calculated by ISDA SIMM may not satisfy the regulatory requirements.

4.1 Pricing Swaptions under the SABR Model

In the numerical experiments we evaluate a European swaption under the SABR model as a case study.

As mentioned above, regulations require derivatives transactions to be centrally cleared. However, transactions with non-linear risks such as options are not subject to the regulations and are traded in OTC because it is difficult for central counterparties to specify a concrete calculation method of margins, similar to regulations in OTC derivatives. European swaptions are commonly traded as a plain vanilla interest rate derivative and are not centrally cleared due to its non-linear risk.

In this option the underlying asset is the forward swap rate X and when the option is exercised at maturity, a swap transaction can be initiated [10] at strike rate K. For simplicity, we normalize the current annuity value to unity. The payoffs of call and put options at maturity T with the strike rate K are given by V^C for a call option and V^P for a put option respectively as follows:

$$V^C(T) = (X(T) - K)^+, \quad V^P(T) = (K - X(T))^+$$

Next, we set the valuation model as the SABR model, a special type of stochastic volatility models, proposed by Hagan et al. (2002). According to Brigo and Mercurio (2001) and Andersen and Piterbarg (2010), this model is commonly used in the interest rate derivatives market to represent the skew of the options market by setting the underlying asset as the forward swap rate.

In this model, the forward swap rate and its volatility satisfy the following stochastic differential equation under the swap measure $\mathbf{Q}$.

$$\begin{aligned} dX(t) &= \sigma(t)X^\beta(t)dW_X^{\mathbf{Q}}(t), \\ d\sigma(t) &= \nu\sigma(t)dW_\sigma^{\mathbf{Q}}(t), \end{aligned}$$

[10] Depending on the contract, options may be settled in cash at expiration.

Here, $W_X^{\mathbf{Q}}$ and $W_\sigma^{\mathbf{Q}}$ are standard Brownian motions with $\rho \in [-1, 1]$ under the measure $\mathbf{Q}$. Furthermore, each parameter is a constant such that $\sigma(0) > 0, \beta \in [0, 1]$ and $\nu > 0$.

The CCR indicators such as Mratio or PFE are originally calculated under $\mathbf{P}$. Therefore, it is necessary to transform the measure from $\mathbf{Q}$ to $\mathbf{P}$.

Here, we assume that $\theta_X(t)$ and $\theta_\sigma(t)$ are given as $(\mathcal{F}_t)$-adapted processes satisfying the following equations.

$$dW_X^{\mathbf{Q}}(t) = dW_X(t) - \theta_X(t)dt, \quad dW_\sigma^{\mathbf{Q}}(t) = dW_\sigma(t) - \theta_\sigma(t)dt.$$

Then, according to Girsanov-Maruyama theorem, we can transform the Brownian motions from $dW_X^{\mathbf{Q}}(t), dW_\sigma^{\mathbf{Q}}(t)$ under $\mathbf{Q}$ to $dW_X(t), dW_\sigma(t)$ under $\mathbf{P}$. In other words, the dynamics of X and σ under $\mathbf{P}$ can be determined.

In the numerical experiments conducted later, we set the drift term under $\mathbf{P}$ in the same way as Kitani and Nakagawa (2024).

$$\mu_X(t) = \theta_X(t)\sigma(t)X^\beta(t), \quad \mu_\sigma(t) = \theta_\sigma(t)\nu\sigma(t).$$

From Hagan et al. (2002) there are two types of approximations for European option price under the SABR model: a Log-Normal type and a Normal type.

The first one, which is closer to the log normal model, is following;

$$\begin{aligned}
V^{C(L)}(T-t, X(t), \sigma_L(t), K) &\simeq X(t)\Phi\left(d_+(t)\right) - K\Phi\left(d_-(t)\right), \\
V^{P(L)}(T-t, X(t), \sigma_L(t), K) &\simeq -X(t)\Phi\left(-d_+(t)\right) + K\Phi\left(-d_-(t)\right).
\end{aligned}$$

where,

$$\begin{aligned}
z_L(t) &:= z_L(X(t), \sigma_L(t), \beta, \nu, K) = \frac{(X(t)K)^{\frac{1-\beta}{2}} \ln \frac{X(t)}{K}}{\sigma(t)}\nu, \\
\mathfrak{X}(z, \rho) &:= \ln\left(\frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho}\right), \\
\Theta_L(t) &:= \Theta_L(X(t), \sigma(t), \beta, \rho, \nu, K) = \frac{1}{24}\frac{\sigma^2(t)\left(1-\beta\right)^2}{\left(X(t)K\right)^{1-\beta}} + \frac{\rho\nu\sigma(t)\beta}{4\left(X(t)K\right)^{\frac{1-\beta}{2}}} + \frac{2-3\rho^2}{24}\nu^2,
\end{aligned}$$

$$\begin{aligned}
\sigma_L(t) &:= \sigma_L(T-t, X(t), \sigma(t), \beta, \rho, \nu, K) \\
&= \frac{\sigma(t)}{(X(t)K)^{\frac{1-\beta}{2}}\left[1 + \frac{(1-\beta)^2}{24}\left(\ln\frac{X(t)}{K}\right)^2 + \frac{(1-\beta)^4}{1920}\left(\ln\frac{X(t)}{K}\right)^4\right]}\frac{z_L(t)}{\mathfrak{X}(z_L(t), \rho)}\left[1 + \Theta_L(t)\left(T-t\right)\right],
\end{aligned}$$

$$d_+(t) := d_+(T-t, X(t), \sigma_L(t), K) = \frac{\ln\frac{X(t)}{K} + \frac{\sigma_L^2(t)(T-t)}{2}}{\sigma_L(t)\sqrt{T-t}}, \quad d_-(t) := d_+(t) - \sigma_L(t)\sqrt{T-t}.$$

The other, which is closer to the normal model, is following;

$$
\begin{aligned}
V^{C(N)}(T-t, X(t), \sigma_N(t), K) &\simeq (X(t)-K)\Phi(d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)),\\
V^{P(N)}(T-t, X(t), \sigma_N(t), K) &\simeq (K-X(t))\Phi(-d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)),
\end{aligned}
$$

where,

$$
\begin{aligned}
z_N(t) &:= z_N(X(t), \sigma(t), \beta, \nu, K) = \frac{\nu}{\sigma(t)} \frac{X(t)^{1-\beta} - K^{1-\beta}}{1-\beta},\\
\mathfrak{X}(z, \rho) &:= \ln\left(\frac{\sqrt{1-2\rho z + z^2} + z - \rho}{1-\rho}\right),\\
\Theta_N(t) &:= \Theta_N(X(t), \sigma(t), \beta, \rho, \nu, K)\\
&= \frac{1}{24}\frac{\sigma^2(t)\beta(\beta-2)(1-\beta)^2\left(\ln\frac{X(t)}{K}\right)^2}{(X^{1-\beta}(t)-K^{1-\beta})^2} + \frac{\rho\nu\sigma(t)}{4}\frac{X^\beta(t)-K^\beta}{X(t)-K} + \frac{2-3\rho^2}{24}\nu^2,\\
\sigma_N(t) &:= \sigma_N(T-t, X(t), \sigma(t), \beta, \rho, \nu, K)\\
&= \frac{\sigma(t)(1-\beta)(X(t)-K)}{X^{1-\beta}(t)-K^{1-\beta}} \frac{z_N(t)}{\mathfrak{X}(z_N(t), \rho)}\left[1 + \Theta_N(t)(T-t)\right],\\
d(t) &:= d(T-t, X(t), \sigma_N(t), K) = \frac{X(t)-K}{\sigma_N(t)\sqrt{T-t}}.
\end{aligned}
$$

From these approximations, we can derive the partial derivatives of V ($V_x, V_\sigma, V_{xx}, V_{\sigma\sigma}$, and $V_{x\sigma}$). [11] As a result, we can calculate the CCR indicators specifically.

4.2 Assumptions for Numerical Experiments

We assume that the swaption strike price is 3% or 5% (OTM) and its option term is $T-t = 1$ Year. [12] The payoff at maturity T are as follows;

$$
V^C(T) = (X(T) - 5\%)^+, \quad V^P(T) = (3\% - X(T))^+.
$$

We set the coefficients of the SABR model. First, we suppose $\beta = 0.75$ because the interest rate level is high and the underlying asset is close to a log normal process with $\beta = 1$. In addition, we set $\nu = 20\%$ for the volatility of volatility and $\rho = 0.5$ for the correlation between the underlying asset and its volatility.

$$
\begin{aligned}
dX(t) &= \sigma(t)X^{0.75}(t)dW_X(t),\\
d\sigma(t) &= 0.20\sigma(t)dW_\sigma(t),
\end{aligned}
$$

[11] For specific derivations, refer to the Appendix.

[12] In the market convention the options with ATM strike are generally traded. However since the underlying asset price changes after the trade, the strikes of almost all transactions are not ATM. In addition, active funds, which hold a large number of options positions, mainly trade OTM, which is low option price and high return. Therefore, we mainly use OTM options in numerical experiments.

$$dW_X(t)dW_\sigma(t) = 0.50dt.$$

The MPoR δ^* is set to 14 days based on Basel Committee on Banking Supervision (BCBS) (2016).

$$\delta^* = \frac{10\text{Business Days}}{1\text{Year}} \simeq \frac{14\text{Days}}{365\text{Days}}.$$

We assume the drifts $\mu_X = 0$ and $\mu_\sigma = 0$ based on Kitani and Nakagawa (2024). These parameter values are chosen to reflect typical market conditions for interest rate swaptions.

In practice, the SIMM coefficients (RW, VRW) have different values at different maturities [13]. In this study, for the sake of simplicity we assume RW= 60 and VRW= 0.20 for all maturities, according to the SIMM coefficients in the regular currencies as shown in Table 1.

Table 1: The SIMM coefficients version 2.7 revised in Dec 2024

type	ccy	2w	1m	3m	6m	1yr	2yr	3yr	5yr	10yr	15yr	20yr	30yr
RW	Regular	109	106	91	69	68	68	66	61	59	59	57	65
	Low-vol	15	21	10	10	11	15	18	23	25	23	23	25
	High-vol	171	102	94	96	105	96	99	93	99	100	101	96
VRW	All	0.20											

We consider four jump scenarios below for the stress test. Accordingly, the expectations of both J_X and J_σ are kept identical across all scenarios excluding No-jump scenario.

- Stochastic
  This scenario is designed to resemble the "1+3 standard" employed in ISDA SIMM. The method estimates the SIMM coefficients by historical data that consists of one stress year, such as the LB cases, plus three non-stress years.
  In this scenario, with 25 % probability underlying asset and its volatility jump like the LB cases with a decline in interest rate minus 1 % and a spike in its implied volatility plus 20 %. With 75 % probability there is no jump.

$$j_1 = (-1.0\%, +20\%), \quad p_1 = 25.0\%,$$
$$j_2 = (\pm 0.0\%, \pm 0\%), \quad p_2 = 75.0\%.$$

- Constant
  This scenario resembles the estimation of the SIMM coefficients. In this scenario the

[13] ISDA SIMM defines three types of RW for each currency (Regular, Low volatility, High volatility). Regular covers USD, EUR, GBP, CHF, AUD, NZD, CAD, SEK, NOK, DKK, HKD, KRW, SGD and TWD. Low volatiloty is only for the JPY, while other currencies are classified as high volatility. VRW is common to all currencies.

jump is not stochastic and equals the expectation of jump in Stochastic scenario. [14]

$$j_1 = (-0.25\%, +5\%), \quad p_1 = 100\%.$$

- Two-Steps
  While Stochastic scenario assumes a jump equal to the LB, larger jumps can occur in reality. Therefore, to incorporate such significant jumps while keeping the occurrence rate and expectation identical to those in Stochastic, we decompose j_1 in Stochastic scenario into j_1 and j_2.

$$\begin{aligned} j_1 &= (-1.5\%, +25\%), \quad p_1 = 12.5\%, \\ j_2 &= (-0.5\%, +15\%), \quad p_2 = 12.5\%, \\ j_3 &= (\pm 0.0\%, \pm 0.0\%), \quad p_3 = 75.0\%. \end{aligned}$$

- No-Jump
  There is no jump.

$$j_1 = (\pm 0.0\%, \pm 0.0\%), \quad p_1 = 100\%.$$

The scenarios and parameters assumed in the numerical experiments in this study are shown in Tables 2 and 3.

4.3 Results of Numerical Experiments

We first verify whether IM satisfies the regulatory requirements or not by evaluating Mratio. If it is 99% or higher, we can confirm that the regulatory requirements are satisfied. But, if not, they are not satisfied and we measure the risk amount by PFE.

In the numerical experiments, we conduct Mratio and PFE for the volatility σ in the range of 7.5% to 22.5%. The same analysis was conducted for the relationship of the underlying asset X horizontally from 1.75% to 6.25%. For these values, X is the central level since the 1990s. Although we cannot observe σ directly, it is roughly considered the central level.

Fig. 3 and 4 show that in No-Jump scenario, since Mratio, depicted by the blue line, stays near the 99% and this means that IM satisfies the regulatory requirements. Conversely, in the other three scenarios, Mratio often falls below 99%, and this indicates IM may not satisfy the regulatory requirements even in low volatility.

Next, we evaluate PFE in these three scenarios. It is observed that PFE in Two-Steps and Stochastic scenarios are larger than those in Constant scenarios. This result stems from the fact that in these two scenarios the impact is stochastic.

Furthermore, PFE remained constant regardless of the level of σ.

On the other hand, Fig. 5 and 6 show that PFE increases as the underlying asset price approaches At The Money (ATM, $X = K$).

[14] ISDA estimates SIMM coefficients as the 99% value of variation in 14 days calculated in four-year data. Therefore, SIMM coefficients may be more conservative than the Constant scenario calculated as an expectation.

Table 2: Scenarios of Jump

Scenario	$J = (J_X, J_\sigma)$	p_n	note
Two-Steps	$(-1.5\%, +25\%)$ $(-0.5\%, +15\%)$ $(\pm 0.0\%, \pm 0.0\%)$	12.5% 12.5% 75.0%	Decomposition of a stochastic into two parts
Stochastic	$(-1.0\%, +20\%)$ $(\pm 0.0\%, \pm 0.0\%)$	25.0% 75.0%	based on 1+3 standard
Constant	$(-0.25\%, +5\%)$	100%	based on the calculations of SIMM coefficients
No-Jump	$(\pm 0.0\%, \pm 0.0\%)$	100%	Nothing

Table 3: Parameters

transaction	K	3%,5% (OTM)
	$T - t$	1.0 Year
pricing model	β	0.75
	ν	20 %
	ρ	0.50
	μ_X, μ_σ	0.00
	δ^*	14 / 365
ISDA SIMM	RW	60
	VRW	0.20

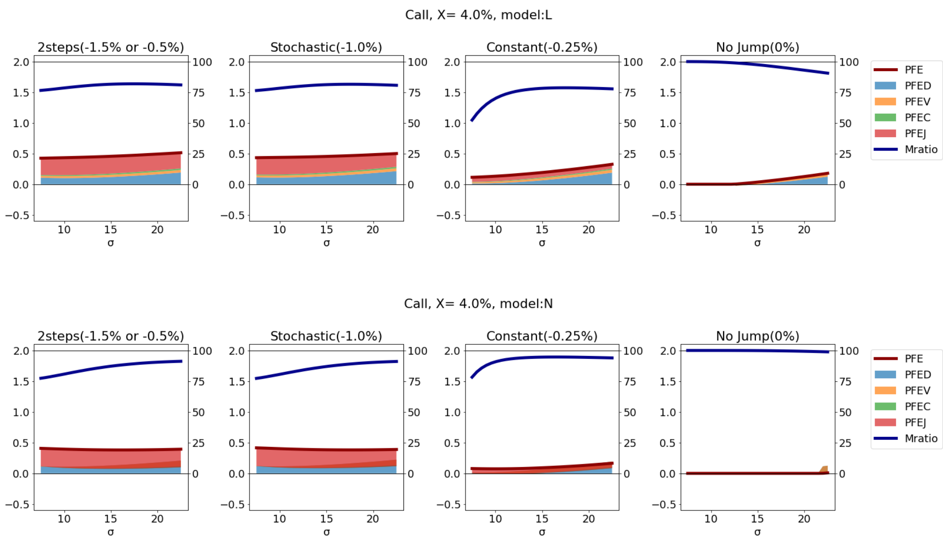

Fig. 3: By scenario of jump, the volatility and CCR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

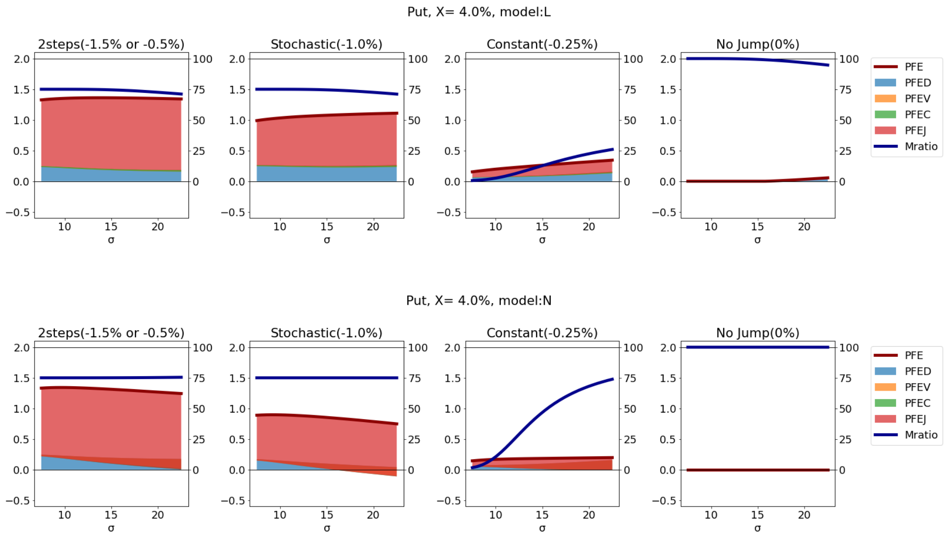

Fig. 4: By scenario of jump, the volatility and CCR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

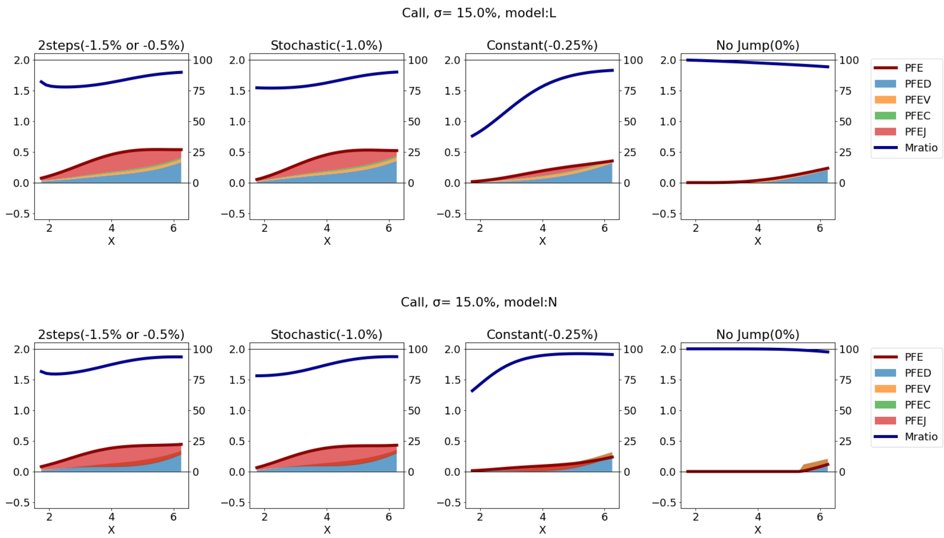

Fig. 5: By scenario of jump, the underlying asset and CCR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

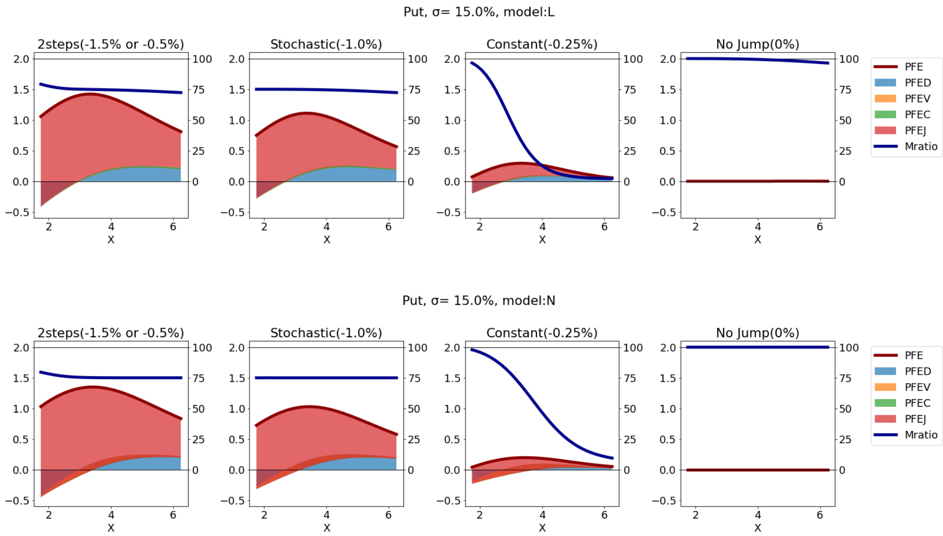

Fig. 6: By scenario of jump, the volatility and CCR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

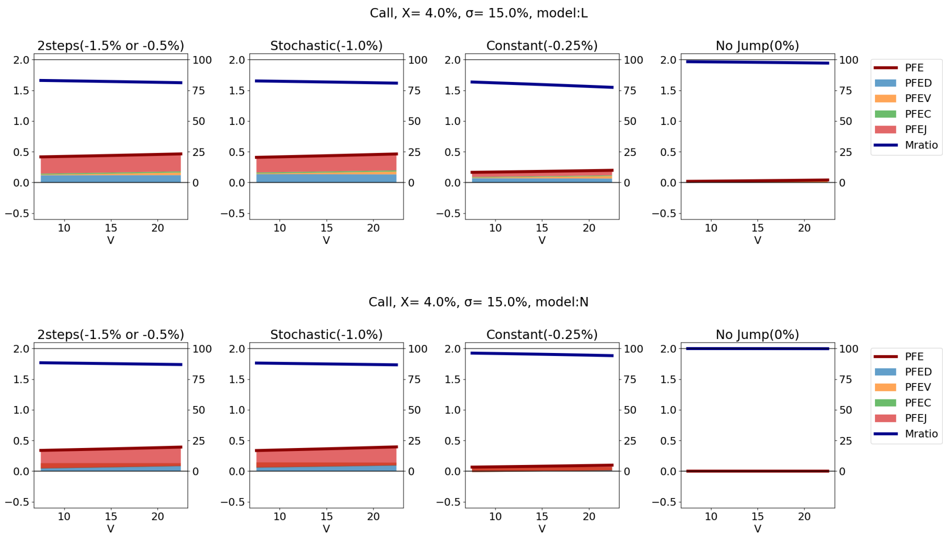

Fig. 7: By scenario of jump, the volatility and CCR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

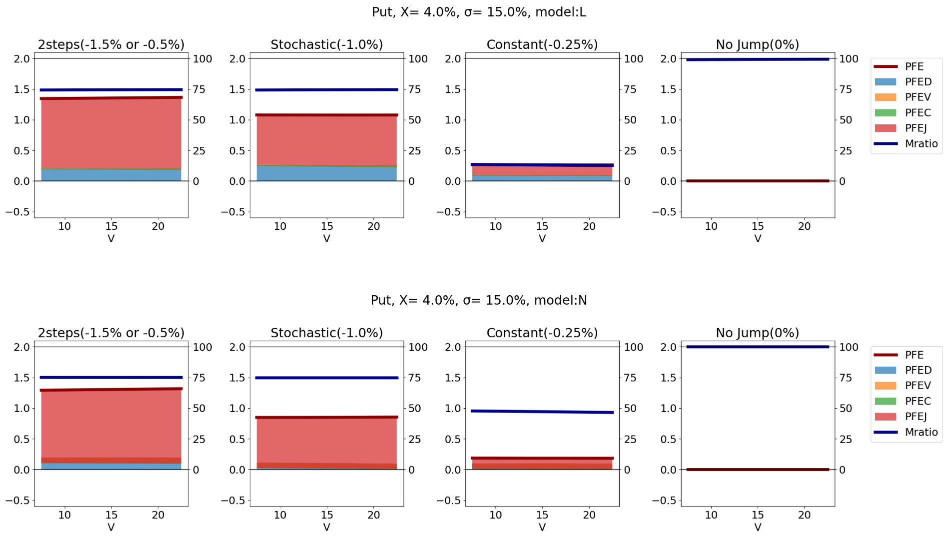

Fig. 8: By scenario of jump, the volatility and CCR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

Subsequently, we compare the differences in PFE between call and put options. In this study we assume the impact is the Lehman Brothers case with a decline in interest rates and a spike in their implied volatility. Therefore, in Put option as both the changes contribute to increase exposure, you can see that PFE of Put option are larger than those of Call option.

Finally, we check the factor PFE. It is evident that the jump factor PFE, depicted by the red area, is the largest. In particular, in Two-Steps and Stochastic scenarios where the impact is stochastic the factor PFE of jump is larger than that in the other scenarios. The results confirm that jump is a significant driver in PFE calculation.

In addition, to assess robustness in a limited sense, we evaluate PFE and Mratio for the different levels of the volatility of volatility ν, as shown in Fig. 7 and 8, and find that the results are broadly in line with the previous experiments.

We conduct numerical experiments with jumps to assess whether IM calculated by ISDA SIMM cover the exposure or not even under the impact of a SIC default. As a result, Mratio clearly falls below 99 % due to the jump.

These results suggest that depending on the transaction type and the nature of the jump under the impact IM calculated by ISDA SIMM may not eliminate CCR to the regulatory level. Furthermore, it is crucial for the measurement and management of CCR to identify the jumps to which certain SIC's transactions are vulnerable and to incorporate these jumps into the model.

We next examine whether MPoR reduction approach can mitigate CCR, as an additional analyses.

5 Limited Effectiveness of MPoR Reduction

This section examines the effectiveness of MPoR reduction as a risk mitigation approach. As a result, the effect of this approach may be limited depending on the transaction type and the nature of the jump.

It is generally known that MPoR (δ^*), the period from default to liquidation, is not uniform, and varies depending on the legal form of the default and the circumstances leading to the default.

Many derivative transactions are conducted based on ISDA Master Agreement developed by International Swaps and Derivatives Association (ISDA) (2002). This agreement refers to some definitions including the event of default (EOD) by International Swaps and Derivatives Association (ISDA) (2006) and International Swaps and Derivatives Association (ISDA) (2021). They also define the procedures for the counterparty defaults.

Andersen et al. (2017) mentions that financial institutions do not immediately initiate the procedures in order to maintain their reputation. As a result of such policies, MPoR may be extended, thereby unintentionally increasing CCR.

On the other hand, they also show that by reducing MPoR financial institutions can mitigate CCR. A concrete method to reduce MPoR is to establish an operational framework for prompt default procedures. From a legal perspective, they can reduce MPoR entering into an amendment agreement bilaterally to refine the definition of EOD to accelerate the triggering of default.

Here, we conduct numerical experiments of the CCR indicators corresponding to a smaller MPoR.

Since the regulatory requirements are satisfied in No-Jump scenario in the previous section, this section examines the other three scenarios.

Fig. 9 and 10 show that PFE decreases due to the MPoR reduction. On the other hand, in Two-Steps and Stochastic scenarios MPoR reduction mitigates PFE of Call options but even when MPoR is reduced to zero, PFE does not necessarily decline to zero. In addition, it does not significantly mitigate PFE of Put options.

In the previous section we see that factor PFE of jump is larger in these scenarios where the impact is stochastic. Since it cannot be eliminated by this approach, this approach cannot eliminate original PFE in these scenarios.

These analyses suggest that the effectiveness of the MPoR reduction approach may be limited depending on the transaction type and the nature of the jump and alternative approaches beyond MPoR reduction are required to mitigate CCR.

6 Conclusion

Traditionally, CCR has been discussed as an issue of risk management within individual financial institutions. Therefore, financial institutions negotiated margin agreement to reflect their original risk management policies on margin calculation and delivery. [15]

[15] Before the margin regulations, an independent amount (IA), which plays the same role as IM, is set. Although IA can be set even now, it cannot substitute for the IM regulations.

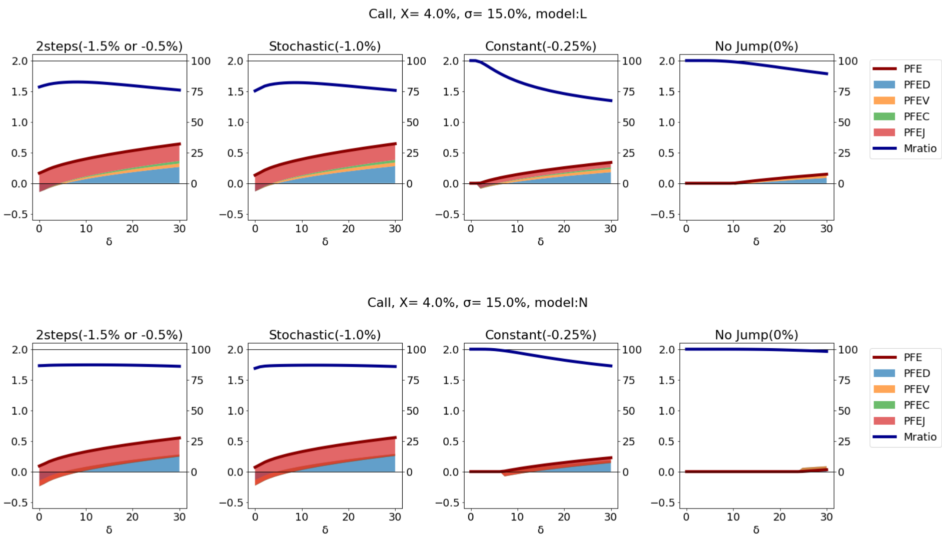

Fig. 9: By scenario of Jump, MPoR and CCR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

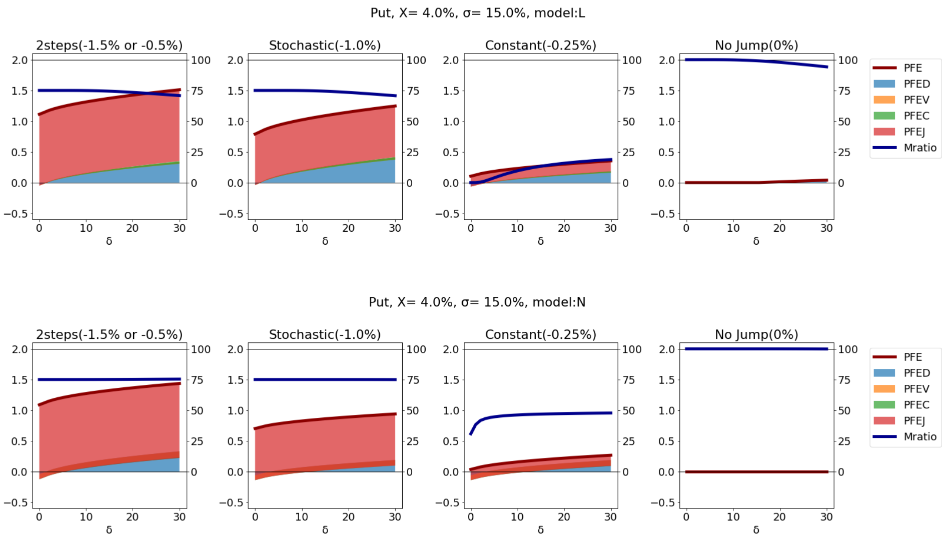

Fig. 10: By scenario of Jump, MPoR and CCR indicators .The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

However, after the financial crisis (2008), the initiative in CCR management has shifted to the regulators, and uniform margin regulations have been implemented. In particular since the mandate of the regulations, IM calculation methods are simplified and unified into ISDA SIMM.

However, it is generally known that a SIC default impacts financial markets. To incorporate the impact, ISDA SIMM adopts the "1+3 standard" as a conservative IM calculation method, nevertheless it does not accurately model the impact.

This study examines whether IM calculated by ISDA SIMM satisfies the regulatory requirements under the impact of a SIC default.

We derive analytical approximate formulass for key CCR indicators such as Mratio and PFE, that directly incorporate the impact of SIC default as discrete jumps, thereby avoiding the need for computationally intensive stochastic simulations. Furthermore, we conduct numerical experiments under several jump scenarios.

The results show that Mratio often falls below 99% in the scenarios with jump, and PFE is larger in the scenarios where the jump is stochastic.

It is verified that depending on the transaction type and the nature of the jump under the impact IM calculated by ISDA SIMM may not eliminate CCR to the regulatory level. Consequently, our findings imply that it is crucial for measurement and management of CCR to accurately model the impact of a SIC default.

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Appendix

A Derivation of the SABR model sensitivity

Two types of approximation formulas for a European option under the SABR model are proposed by Hagan et al. (2002), namely, the Log Normal type and Normal type. In this appendix, we derive the partial derivatives of each approximation formula.

A.1 Log Normal type

The approximation formula of Log Normal type is shown as follows:

$$V^{C(L)}(T-t, X(t), \sigma_L(t), K) \simeq X(t)\Phi\left(d_+(t)\right) - K\Phi\left(d_-(t)\right),$$
$$V^{P(L)}(T-t, X(t), \sigma_L(t), K) \simeq -X(t)\Phi\left(-d_+(t)\right) + K\Phi\left(-d_-(t)\right).$$

where,

$$z_L(t) := z_L(X(t), \sigma_L(t), \beta, \nu, K) = \frac{(X(t)K)^{\frac{1-\beta}{2}} \ln \frac{X(t)}{K}}{\sigma(t)}\nu,$$

$$\mathfrak{X}(z,\rho) := \ln\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right),$$

$$\Theta_L(t) := \Theta_L(X(t),\sigma(t),\beta,\rho,\nu,K) = \frac{1}{24}\frac{\sigma^2(t)\,(1-\beta)^2}{(X(t)K)^{1-\beta}} + \frac{\rho\nu\sigma(t)\beta}{4\,(X(t)K)^{\frac{1-\beta}{2}}} + \frac{2-3\rho^2}{24}\nu^2,$$

$$\begin{aligned}\sigma_L(t) &:= \sigma_L(T-t,X(t),\sigma(t),\beta,\rho,\nu,K)\\ &= \frac{\sigma(t)}{(X(t)K)^{\frac{1-\beta}{2}}\left[1+\frac{(1-\beta)^2}{24}\left(\ln\frac{X(t)}{K}\right)^2+\frac{(1-\beta)^4}{1920}\left(\ln\frac{X(t)}{K}\right)^4\right]}\frac{z_L(t)}{\mathfrak{X}(z_L(t),\rho)}\left[1+\Theta_L(t)\,(T-t)\right],\end{aligned}$$

$$d_+(t) := d_+(T-t,X(t),\sigma_L(t),K) = \frac{\ln\frac{X(t)}{K}+\frac{\sigma_L^2(t)(T-t)}{2}}{\sigma_L(t)\sqrt{T-t}}, \quad d_-(t) := d_+(t)-\sigma_L(t)\sqrt{T-t}.$$

Before proceeding with determination of the partial derivatives, we first derive the relevant formulas for the standard normal distribution.

$$X(t)\phi\left(d_+(t)\right) = X(t)\frac{1}{\sqrt{2\pi}}e^{-\frac{d_+^2(t)}{2}} = X(t)\frac{1}{\sqrt{2\pi}}e^{-\frac{d_-^2(t)}{2}-\ln\frac{X(t)}{K}} = K\phi\left(d_-(t)\right),$$

$$-\frac{d_+^2(t)}{2} = -\frac{1}{2}\left(\frac{\ln\frac{X(t)}{K}}{\sigma_L(t)\sqrt{T-t}}-\frac{\sigma_L(t)\sqrt{T-t}}{2}\right)^2-\ln\frac{X(t)}{K} = -\frac{d_-^2(t)}{2}-\ln\frac{X(t)}{K},$$

$$\frac{\partial d_+}{\partial y}(t) = \frac{\partial}{\partial y}\left(\frac{\ln\frac{X(t)}{K}}{\sigma_L(t)\sqrt{T-t}}-\frac{\sigma_L(t)\sqrt{T-t}}{2}\right)+\sqrt{T-t}\sigma_{L,y}(t) = \frac{\partial d_-}{\partial y}(t)+\sqrt{T-t}\sigma_{L,y}(t).$$

Using these formulas, Delta (V_x), Vega (V_σ) and Theta (V_t) can be derived as follows,

$$\begin{aligned}V_x^{C(L)}(t) &= \Phi\left(d_+(t)\right)+K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,x}(t),\\ V_x^{P(L)}(t) &= -\Phi\left(-d_+(t)\right)+K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,x}(t),\\ V_\sigma^{C(L)}(t) &= V_\sigma^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,\sigma}(t),\\ V_t^{C(L)}(t) &= V_t^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,t}(t).\end{aligned}$$

Also, Gamma (V_{xx}), Volga ($V_{x\sigma}$) and Vanna ($V_{\sigma\sigma}$) can be derived as follows.

$$V_{xx}^{C(L)}(t) = V_{xx}^{P(L)}(t) = \phi\left(d_+(t)\right)\frac{\partial d_+}{\partial X}(t)+K\phi\left(d_-(t)\right)\sqrt{T-t}\left(\sigma_{L,xx}(t)-d_-\frac{\partial d_-}{\partial X}(t)\sigma_{L,x}(t)\right),$$

$$V_{x\sigma}^{C(L)}(t) = V_{x\sigma}^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\left(\sigma_{L,x\sigma}(t) - d_-\frac{\partial d_-}{\partial X}(t)\sigma_{L,\sigma}(t)\right),$$
$$V_{\sigma\sigma}^{C(L)}(t) = V_{\sigma\sigma}^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\left(\sigma_{L,\sigma\sigma}(t) - d_-\frac{\partial d_-}{\partial \sigma}(t)\sigma_{L,\sigma}(t)\right).$$

where,

$$\frac{\partial d_+}{\partial X}(t) = \frac{1}{X(t)\sigma_L(t)\sqrt{T-t}} - \frac{d_-(t)}{\sigma_L(t)}\sigma_{L,x}(t), \quad \frac{\partial d_+}{\partial \sigma}(t) = -\frac{d_-(t)}{\sigma_L(t)}\sigma_{L,\sigma}(t),$$
$$\frac{\partial d_-}{\partial X}(t) = \frac{1}{X(t)\sigma_L(t)\sqrt{T-t}} - \frac{d_+(t)}{\sigma_L(t)}\sigma_{L,x}(t), \quad \frac{\partial d_-}{\partial \sigma}(t) = -\frac{d_+(t)}{\sigma_L(t)}\sigma_{L,\sigma}(t).$$

Note that, the function σ_L and θ_L takes a singular point at $X = K$.

$$\sigma_L(t) = \frac{\sigma(t)}{X^{1-\beta}(t)}\left[1 + \theta_L(t)\left(T-t\right)\right],$$
$$\theta_L(t) = \frac{1}{24}\frac{\sigma^2(t)\left(1-\beta\right)^2}{X^{2(1-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta}{4X^{1-\beta}(t)} + \frac{2-3\rho^2}{24}\nu^2.$$

In the case of $X = K$, the partial derivatives of the functions σ_L and θ_L can be derived by:

$$\sigma_{L,x}(t) = \frac{\sigma(t)\left(\beta-1\right)}{X^{2-\beta}(t)}\left[1 + \theta_L\left(T-t\right)\right] + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,x}(t)\left(T-t\right),$$
$$\sigma_{L,\sigma}(t) = \frac{1}{X^{1-\beta}(t)}\left[1 + \theta_L(t)\left(T-t\right)\right] + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,\sigma}\left(T-t\right),$$
$$\sigma_{L,t}(t) = -\frac{\sigma(t)}{X^{1-\beta}(t)}\theta_L(t),$$
$$\sigma_{L,xx}(t) = \frac{\sigma(t)\left(\beta-1\right)\left(\beta-2\right)}{X^{3-\beta}(t)}\left[1 + \theta_L(t)\left(T-t\right)\right]$$
$$+ 2\frac{\sigma(t)\left(\beta-1\right)}{X^{2-\beta}(t)}\theta_{L,x}(t)\left(T-t\right) + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,xx}(t)\left(T-t\right),$$
$$\sigma_{L,x\sigma}(t) = \frac{\left(\beta-1\right)}{X^{2-\beta}(t)}\left[1 + \theta_L(t)\left(T-t\right)\right] + \frac{1}{X^{1-\beta}(t)}\theta_{L,x}(t)\left(T-t\right)$$
$$+ \frac{\sigma(t)\left(\beta-1\right)}{X^{2-\beta}(t)}\theta_{L,\sigma}(t)\left(T-t\right) + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,x\sigma}(t)\left(T-t\right),$$
$$\sigma_{L,\sigma\sigma}(t) = 2\frac{1}{X^{\beta-1}(t)}\theta_{L,\sigma}(t)\left(T-t\right) + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,\sigma\sigma}(t)\left(T-t\right),$$
$$\theta_{L,x}(t) = \frac{1}{12}\frac{\sigma^2(t)\left(\beta-1\right)^3}{X^{3-2\beta}(t)} + \frac{\rho\nu\sigma(t)\beta\left(\beta-1\right)}{4X^{2-\beta}(t)},$$
$$\theta_{L,\sigma}(t) = \frac{1}{12}\frac{\sigma(t)\left(\beta-1\right)^2}{X^{2-2\beta}(t)} + \frac{\rho\nu\beta}{4X^{1-\beta}(t)},$$

$$\theta_{L,xx}(t) = \frac{1}{12}\frac{\sigma^2(t)\left(\beta-1\right)^3\left(2\beta-3\right)}{X^{2(2-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta\left(\beta-1\right)\left(\beta-2\right)}{4X^{3-\beta}(t)},$$
$$\theta_{L,x\sigma}(t) = \frac{1}{6}\frac{\sigma(t)\left(\beta-1\right)^3}{X^{3-2\beta}(t)} + \frac{\rho\nu\beta\left(\beta-1\right)}{4X^{2-\beta}(t)},$$
$$\theta_{L,\sigma\sigma}(t) = \frac{1}{12}\frac{\left(\beta-1\right)^2}{X^{2(1-\beta)}(t)}.$$

A.2 Normal type

The approximation formula of Normal type is also described as follows:

$$V^{C(N)}(T-t, X(t), \sigma_N(t), K) \simeq (X(t)-K)\Phi(d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)),$$
$$V^{P(N)}(T-t, X(t), \sigma_N(t), K) \simeq (K-X(t))\Phi(-d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)).$$

where,

$$z_N(t) := z_N(X(t), \sigma(t), \beta, \nu, K) = \frac{\nu}{\sigma(t)}\frac{X(t)^{1-\beta} - K^{1-\beta}}{1-\beta},$$
$$\mathfrak{X}(z,\rho) := \ln\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right),$$
$$\begin{aligned}\Theta_N(t) &:= \Theta_N(X(t), \sigma(t), \beta, \rho, \nu, K)\\ &= \frac{1}{24}\frac{\sigma^2(t)\beta(\beta-2)(1-\beta)^2\left(\ln\frac{X(t)}{K}\right)^2}{(X^{1-\beta}(t)-K^{1-\beta})^2} + \frac{\rho\nu\sigma(t)}{4}\frac{X^{\beta}(t)-K^{\beta}}{X(t)-K} + \frac{2-3\rho^2}{24}\nu^2,\end{aligned}$$
$$\begin{aligned}\sigma_N(t) &:= \sigma_N(T-t, X(t), \sigma(t), \beta, \rho, \nu, K)\\ &= \frac{\sigma(t)(1-\beta)(X(t)-K)}{X^{1-\beta}(t)-K^{1-\beta}}\frac{z_N(t)}{\mathfrak{X}(z_N(t),\rho)}\left[1+\Theta_N(t)(T-t)\right],\end{aligned}$$
$$d(t) := d(T-t, X(t), \sigma_N(t), K) = \frac{X(t)-K}{\sigma_N(t)\sqrt{T-t}}.$$

According to Kitani and Nakagawa (2024),

$$\frac{\partial}{\partial y}\phi\left(d(t)\right) = -\frac{X(t)-K}{\sigma_N(t)\sqrt{T-t}}\phi\left(d(t)\right)\frac{\partial d}{\partial y}(t).$$

Thus, we can derive Delta (V_x), Vega (V_σ) and Theta (V_t) as follows,

$$V_x^C(t) = \Phi\left(d(t)\right) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,x}(t),$$
$$V_x^P(t) = -\Phi\left(-d(t)\right) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,x}(t),$$
$$V_\sigma^C(t) = V_\sigma^P(t) = \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,\sigma}(t),$$
$$V_t^C(t) = V^Pt(t) = \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,t}(t) - \frac{\sigma_N\phi\left(d(t)\right)}{2\sqrt{T-t}}.$$

From Kitani and Nakagawa (2024),

$$\frac{\partial}{\partial y}\phi(d) = -d\frac{\partial d}{\partial y}\phi(d),$$
$$\frac{\partial d}{\partial X}(t) = \frac{1}{\sigma_N(t)\sqrt{T-t}}\left(1 - \frac{X(t)-K}{\sigma_N(t)}\sigma_{N,x}(t)\right), \quad \frac{\partial d}{\partial \sigma}(t) = -\frac{X(t)-K}{\sigma_N^2(t)\sqrt{T-t}}\sigma_{N,\sigma}(t).$$

Using the formula below,

$$\frac{\partial}{\partial X}\left(-\Phi(-d)\right) = \frac{\partial}{\partial X}\Phi(d)$$

In this case, Gamma (V_{xx}), Volga ($V_{x\sigma}$) and Vanna ($V_{\sigma\sigma}$) can be derived as follows,

$$V_{xx}^C(t) = V_{xx}^P(t) = \left(\frac{X(t)-K}{\sigma_N(t)}\sigma_{N,x}(t) - 1\right)^2 \frac{\phi\left(d(t)\right)}{\sigma_N(t)\sqrt{T-t}} + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,xx}(t),$$

$$\begin{aligned} V_{x\sigma}^C(t) &= V_{x\sigma}^P(t) \\ &= \left(\frac{X(t)-K}{\sigma_N(t)}\sigma_{N,x}(t) - 1\right)\phi\left(d(t)\right)\frac{X(t)-K}{\sigma_N^2(t)\sqrt{T-t}}\sigma_{N,\sigma}(t) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,x\sigma}(t), \end{aligned}$$

$$V_{\sigma\sigma}^C(t) = V_{\sigma\sigma}^P(t) = \frac{(X(t)-K)^2}{\sigma_N^3(t)\sqrt{T-t}}\phi\left(d(t)\right)\sigma_{N,\sigma}^2(t) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,\sigma\sigma}(t).$$

Same as the Log Normal type, the functions σ_N and θ_N also take a singular point at $X = K$.

$$\begin{aligned} \sigma_N(t) &= \sigma(t)X^\beta(t)\left[1 + \theta_N(t)\left(T-t\right)\right], \\ \theta_N(t) &= \frac{1}{24}\frac{\sigma^2(t)\beta\left(\beta-2\right)}{X^{2(1-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta}{4X^{1-\beta}(t)} + \frac{2-3\rho^2}{24}\nu^2, \end{aligned}$$

In the case of $X = K$, the partial derivatives of the functions σ_L and θ_L can be derived by:

$$\begin{aligned} \sigma_{N,x}(t) &= \sigma(t)\beta X^{\beta-1}(t)\left[1 + \theta_N\left(T-t\right)\right] + \sigma(t)X^\beta(t)\theta_{N,x}(t)\left(T-t\right), \\ \sigma_{N,\sigma}(t) &= X^\beta(t)\left[1 + \theta_N(t)\left(T-t\right)\right] + \sigma(t)X^\beta(t)\theta_{N,\sigma}\left(T-t\right), \\ \sigma_{N,t}(t) &= -\sigma(t)X^\beta(t)\theta_N(t), \\ \sigma_{N,xx}(t) &= \sigma(t)\beta\left(\beta-1\right)X^{\beta-1}(t)\left[1 + \theta_L(t)\left(T-t\right)\right] + \sigma(t)\beta X^{\beta-1}(t)\theta_{L,x}(t)\left(T-t\right) \\ &\quad + \sigma(t)\beta X^{\beta-1}\theta_{L,x}(t) + \sigma(t)X^\beta(t)\theta_{L,xx}(t)\left(T-t\right), \\ \sigma_{N,x\sigma}(t) &= \beta X^{\beta-1}(t)\left[1 + \theta_N(t)\left(T-t\right)\right] + X^\beta(t)\theta_{N,x}(t)\left(T-t\right) \end{aligned}$$

$$
\begin{aligned}
\sigma_{N,\sigma\sigma}(t) = {} & 2X^{\beta}(t)\theta_{N,\sigma}(t)\,(T-t) + \sigma(t)X^{\beta}(t)\theta_{N,\sigma\sigma}(t), \\
& + \sigma(t)\beta X^{\beta-1}(t)\theta_{N,\sigma}(t)\,(T-t) + \sigma(t)X^{\beta}(t)\theta_{N,x\sigma}(t)\,(T-t)\,.
\end{aligned}
$$

where,

$$
\begin{aligned}
\theta_{N,x}(t) &= \frac{1}{12}\frac{\sigma^2(t)\,(\beta-1)\,\beta\,(\beta-2)}{X^{3-2\beta}(t)} + \frac{\rho\nu\sigma(t)\beta\,(\beta-1)}{4X^{2-\beta}(t)}, \\
\theta_{N,\sigma}(t) &= \frac{1}{12}\frac{\sigma(t)\beta\,(\beta-2)}{X^{2(1-\beta)}(t)} + \frac{\rho\nu\beta}{4X^{1-\beta}(t)}, \\
\theta_{N,xx}(t) &= \frac{1}{12}\frac{\sigma^2(t)\,(\beta-1)\,\beta\,(\beta-2)\,(2\beta-3)}{X^{2(2-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta\,(\beta-1)\,(\beta-2)}{4X^{3-\beta}(t)}, \\
\theta_{N,x\sigma}(t) &= \frac{1}{6}\frac{\sigma(t)\beta\,(\beta-1)\,(\beta-2)}{X^{3-2\beta}(t)} + \frac{\rho\nu\beta\,(\beta-1)}{4X^{2-\beta}(t)}, \\
\theta_{N,\sigma\sigma}(t) &= \frac{1}{12}\frac{\beta\,(\beta-2)}{X^{2(1-\beta)}(t)}.
\end{aligned}
$$

Additionally, the functions σ_N and z_N of the Normal type also take a singular point at $\beta = 1$.

$$
\begin{aligned}
\sigma_N(t) &= \frac{\nu(X(t)-K)}{\mathfrak{X}(z_N,\rho)}\left[1+\theta_N(t)\,(T-t)\right], \\
z_N(t) &= \frac{\nu}{\sigma}\ln\frac{X(t)}{K}, \quad \theta_N(t) = -\frac{\sigma^2(t)}{24} + \frac{\rho\nu\sigma(t)}{4} + \frac{2-3\rho^2}{24}\nu^2.
\end{aligned}
$$

B Conditional Expectation

For simplicity, we denote PE_D and PE_V in equation (8) by X_1 and X_2, respectively. We assume that X_1 and X_2 follow a normal distribution with means μ_1 and μ_2, variances σ_1^2 and σ_2^2. Each X_i can be expressed in terms of standard normal variables Z_i as:

$$
X_i = \mu_i + \sigma_i Z_i \quad i = 1, 2.
$$

The correlation coefficient between X_1 and X_2 (equivalently between Z_1 and Z_2) is denoted by ρ. Then, $\mathrm{PE}_Z = \mathrm{PE}_D + \mathrm{PE}_V$ can be expressed as,

$$
Y = X_1 + X_2.
$$

Based on this representation, we derive the expectation μ_Y and variance σ_Y^2 of Y [16] and the correlation coefficient $\rho_{1,Y}$ between X_1 and Y, as follows.

$$
\begin{aligned}
& Y = \mu_Y + \sigma_Y Z_Y, \\
& \mu_Y = \mu_1 + \mu_2, \quad \sigma_Y^2 = \sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2,
\end{aligned}
$$

[16] Y follows a normal distribution from the reproducibility of normal distribution.

$$\rho_{1,Y} = \frac{\mathrm{Cov}\left(X_1, Y\right)}{\sigma_1 \sigma_Y} = \frac{\mathrm{Cov}\left(X_1, X_1 + X_2\right)}{\sigma_1 \sigma_Y} = \frac{\sigma_1^2 + \rho\sigma_1\sigma_2}{\sigma_1 \sigma_Y}.$$

Then we calculate the following conditional expectation.

$$\begin{aligned} \mathbf{E}\left[X_1 \,\middle|\, X_1 + X_2 = A\right] &= \mathbf{E}\left[\mu_1 + \sigma_1 Z_1 \,\middle|\, Y = A\right] \\ &= \mu_1 + \sigma_1 \mathbf{E}\left[Z_1 \,\middle|\, \mu_Y + \sigma_Y Z_Y = A\right], \end{aligned}$$

where A is a constant which represents PFE_Z.

Here, using a random variable Z which is independent of Z_Y, Z_1 is expressed as,

$$Z_1 = \rho_{1,Y} Z_Y + \sqrt{1 - \rho_{1,Y}^2} Z.$$

Then,

$$\begin{aligned} \mathbf{E}\left[Z_1 \,\middle|\, \mu_Y + \sigma_Y Z_Y = A\right] &= \mathbf{E}\left[\rho_{1,Y} Z_Y + \sqrt{1 - \rho_{1,Y}^2} Z \,\middle|\, Z_Y = \frac{A - \mu_Y}{\sigma_Y}\right] \\ &= \rho_{1,Y} \frac{1}{\sigma_Y}\left(A - \mu_Y\right) = \frac{\sigma_1^2 + \rho\sigma_1\sigma_2}{\sigma_1 \sigma_Y^2}\left(A - \mu_Y\right). \end{aligned}$$

Finally, the equation is calculated.

$$\mathbf{E}\left[X_1 \,\middle|\, X_1 + X_2 = A\right] = \mu_1 + \frac{\sigma_1^2 + 2\rho\sigma_2}{\sigma_Y^2}\left(A - \mu_Y\right).$$

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