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Generally, counterparty risk (CPR) in Over-the-Counter (OTC) derivatives has been significantly mitigated by margin regulations. Specifically, Initial Margin (IM) is expected to cover the increment of the exposure during the period from a counterparty default to the actual liquidation. Since margins are delivered only up to the default time, this incremental exposure must be treated as a random variable.

Therefore to calculate IM amount now many financial institutions use ISDA Standard Initial Margin Model (ISDA SIMM) as a unified simplified calculation method.

However, it is generally known that a default of a large financial institution, which is defined as a "Systemically Important Counterparty (SIC)" by Pykhtin and Sokol (2013), impacts financial markets and this impact increases the CPR exposure.

ISDA SIMM attempts to incorporate such events in simplified methods called the "1+3 standard". This method is conservative, but it does not accurately model the impacts.

This study investigates whether IM calculated by ISDA SIMM adequately covers the CPR exposure under the impact of the SIC default.

We model the impact as a jump in risk factors of derivatives within a generalized stochastic volatility model and theoretically derive approximate formulas for some CPR indicators, such as Mratio and PFE. Finally, we conduct numerical experiments

*The views and opinions expressed in this paper are those of the author and do not necessarily reflect those of the organization he belongs to.

across several jump scenarios, using a European swaption under the SABR model as a case study.

The results suggest that, depending on the transaction type and the nature of the jump, IM may not satisfy the regulatory requirements under the impact.

1 Introduction

Counterparty risk (CPR) in Over-the-Counter (OTC) derivatives is the credit risk that arises when a counterparty defaults while a derivative value is positive, the value cannot be realized.

The mandate of the Central Clearing and the Margin Regulations has significantly reduced CPR globally after the financial crisis (2008). The central clearing reduced the amount of OTC derivatives and margin regulations mitigate exposure in OTC derivatives by requiring Variation Margin (VM) and Initial Margin (IM).

In margin regulations the CPR exposure is the value of derivatives at the liquidation time. VM is required to cover the exposure at the default time and IM is required to cover the increments of the value during the Margin Period of Risk (MPoR) from default to liquidation. Since margins are delivered only until the default, at the default time, the increment is not predictable and must be treated as a random variable. For this reason, we need to calculate IM as a stochastic problem at the last date of margin delivery.

To calculate the IM amount we need the model of value process and the estimation of their parameters. Although the regulations require IM to cover 99% of this increment, the regulations do not prescribe a specific calculation method. Therefore, the International Swaps and Derivatives Association (ISDA) developed ISDA Standard Initial Margin Model (ISDA SIMM) as a unified simplified calculation model.

The regulations were adopted in 2016. However, Bank of England (2023) has recommended further refinement of MPoR risk management due to the large losses associated with the default of Archegos Capital Management (ACM, 2021). [1] In response, International Swaps and Derivatives Association (ISDA) (2023) has increased the frequency of updating the coefficients used in ISDA SIMM.

In such a discussion, Kitani and Nakagawa (2024) mention that IM calculated by ISDA SIMM covers 99% of the increment in normal conditions when financial markets are not stressed, but it may not satisfy the regulatory requirements when volatility rises.

It is generally known that the default of large financial institutions impacts the financial markets. Taking the default of Lehman Brothers (LB, 2008) as an example Pykhtin and Sokol (2013) define a "Systemically Important Counterparty (SIC)" as a counterparty whose default would impact financial markets and point out that this impact may increase the CPR exposure.

Additionally, we can observe the actual default cases after LB. For instance the default of small financial institutions like Silicon Valley Bank (SVB, 2023) and Signature Bank (SB, 2023) impacted financial markets. Conversely, the SIC default such as ACM

[1] Bartholomew (2022) mentions that since ACM which was not subject to the regulations did not deliver IM calculated by ISDA SIMM the recommendation is not on point. She also refers, however, that if ACM had delivered IM calculated by ISDA SIMM the exposure would not have been covered by IM.

(2021) did not significantly impact the financial markets as the participants remained calm.

These examples show that it is difficult to predict the impact before the credit event, and the impact is probabilistic.

The ISDA SIMM indirectly attempts to incorporate the impact by the conservative calculation method called the "1+3 standard". However, the method does not model the uncertainty of the impacts.

This study examines whether IM calculated by ISDA SIMM adequately covers the CPR exposure or not under the impact of the counterparty default.

First, we review the actual impacts in some financial institutions default cases. Next, we model the impact as a jump in risk factors within a generalized stochastic volatility model and theoretically derive approximate formulas for some CPR indicators. Finally, we conduct numerical experiments under several jump scenarios, using a European swaption under the SABR model as an example.

The results suggest that, depending on the transaction type and the nature of the jump, IM may not satisfy the regulatory requirements under the impact.

2 Modeling the impact on Financial Markets due to the default of a counterparty

In this section, we review cases of the SICs defaults in recent years and its impact on financial markets. Then, we define a mathematical model for the general CPR exposure and introduce the impact as a jump in risk factors.

2.1 Default of SICs and the impact on financial markets

In 2008, following the default of several SICs, Summit of Financial Market and the World Economy (G20) (2009) decided to implement several regulations to reduce the CPR. Then, central clearing of standard derivatives and the delivery of VM and IM in non-cleared derivatives were mandated.

In particular, the default of LB significantly impacted financial markets, and it has become clear that transactions with SICs involve not only individual credit risk but also systemic risk.

Pykhtin and Sokol (2013) define SIC and this impact may increase the CPR exposure through the jump in risk factors of derivatives. [2]

In fact, the default of Long Term Capital Management (LTCM, 1998) and LB impacted financial markets, as shown in Fig. 1, with a decline in the interest rate and a spike in their implied volatility.

However, in recent years, we can observe the similar default cases.

For instance, the default of ACM, which is a large fund and according to Bouveret and Haferkor (2022) its default inflicted substantial losses upon many G-SIBs, did not impact the financial markets, as shown in Fig. 2. On the other hand, the default of SVB and SB, which are non-GSIBs and only regional banks in the U.S., impacted

[2] Pykhtin and Sokol (2013) introduce this concept as being slightly broader than G-SIBs.

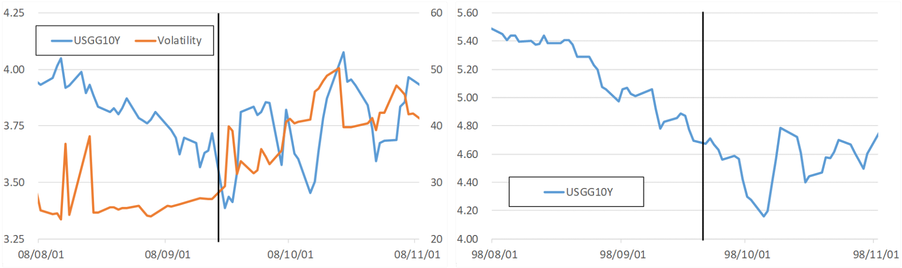

Fig. 1: Financial markets at the default time of LB (Left) and LTCM (Right). The vertical black line indicates the default date.

the financial markets with a decline in the interest rate and a spike in their implied volatility, as shown in Fig. 3

These examples demonstrate that the impact is probabilistic and motivate our modeling of SIC default as a probabilistic jump in risk factors.

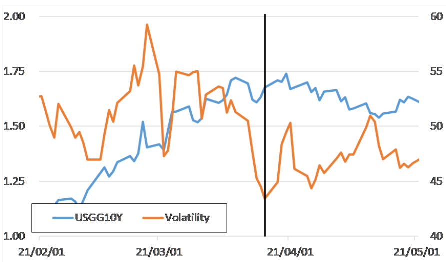

Fig. 2: Financial markets at the default time of ACM.
The vertical black line indicates the default date.

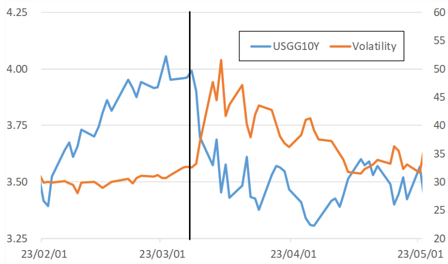

Fig. 3: Financial markets at the default time of SVB and SB.
The vertical black line indicates the default date.

In the next section, we define a general mathematical model and the CPR exposure.

2.2 Counterparty risk and Initial Margin

In this study, we introduce a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ assuming that the probability measure $\mathbf{P}$ is physical and denote the value process of derivatives with maturity T in the continuous period $[0, T]$ as V and the SIC default time as the stopping time τ. Furthermore, we assume that V can be calculated using the underlying asset price X and its volatility σ, and that the filtration $(\mathcal{F}_t)_{t\in[0,T]}$ provides information generated by all random variables excluding the impact of the counterparty default.

$$V(t) = V(t, X(t), \sigma(t))$$

In general, V should incorporate counterparty credit risk. However, in this study, since V is used as the CPR exposure at the SIC default time, we model V excluding credit risk such as Credit Valuation Adjustment (CVA).

In derivatives, CPR arises in derivatives as the credit risk that if counterparty defaults while the value of the derivatives is positive ($V > 0$), we cannot receive the value.

Here, if the counterparty defaults at time t, the value of the derivative continues to change stochastically until the liquidation at time $t + \delta^*$. We denote this period as δ^* (the Margin Period of Risk, MPoR). To mitigate this risk, regulations require OTC derivatives to deliver two types of margin, VM and IM.

In this study, if the counterparty defaults at time t, the margin that should be calculated and delivered at time t is treated as being processed at time $t-$.

Therefore, the CPR exposure is expressed as the Positive Exposure (PE) defined as the value after liquidation minus the pre-default value and IM.

In this study since we calculate the CPR at default time, PE is calculated under the condition of $\tau = t$.

$$\mathrm{PE}(t) = \left(V(\tau + \delta^*) - \mathrm{VM}(\tau-) - \mathrm{IM}(\tau-)\right)^+ \Big|_{\tau=t} . \tag{1}$$

For the sake of simplicity, unless otherwise specified, the following discussion will be based on the condition $\tau = t$, and such specifications will be omitted.

VM directly covers the value V at the counterparty default time, and can be expressed as,

$$\mathrm{VM}(t-) = V(t-) = \lim_{s \to t} V(s).$$

After the default the credit exposure V changes stochastically until the liquidation. Although we need to cover $V(t + \delta^*)$, VM is delivered until the time t. To cover the increment of the value V from default to liquidation, the regulations require IM. IM covers the increment of exposure in MPoR.

$$\begin{aligned} V(t + \delta^*) - \mathrm{VM}(t-) &= V(t + \delta^*) - V(t-) \\ &= V(t + \delta^*) - V(t) + (V(t) - V(t-)) , \end{aligned}$$

But, we need to calculate and deliver IM at time $t-$ so, $V(t + \delta^*)$ is not predictable at the default time $t-$. The regulations require IM to cover 99% of this stochastic increment.

$$\mathrm{IM}^{Reg}(t-) = \mathrm{ess.inf} \left\{ y \in \mathbb{R} \;\middle|\; \mathbf{P}\left(V(\tau + \delta^*) - V(\tau-) \leq y \;\middle|\; \mathcal{F}_\tau, \tau = t \right) \leq 1\% \right\} .$$

To calculate IM amount as required by regulations, we need the model of value process and the estimation of their parameters. However, since they depend on the academic research progress and the management policies of financial institutions, regulations avoid to specify a concrete calculation method. Therefore International

Swaps and Derivatives Association (ISDA) (2024) developed ISDA SIMM as a unified simplified calculation method.

In this method IM is calculated using the sensitivity calculated by each financial institution and the SIMM coefficients (RW and VRW, corresponding to the volatility of each risk factor) estimated from historical data published by ISDA. The outline of the calculation formula is as follows. [3]

$$\mathrm{IM}(t) = \mathrm{IM}_{\mathrm{Delta}}(t) + \mathrm{IM}_{\mathrm{Vega}}(t) + \mathrm{IM}_{\mathrm{Curvature}}(t),$$

where,

$$\begin{aligned}
&\mathrm{IM}_{\mathrm{Delta}}(t) = \mathrm{RW} \cdot V_x(t, X(t), \sigma(t)) \cdot 1\mathrm{bp},\\
&\mathrm{IM}_{\mathrm{Vega}}(t) = \mathrm{VRW} \cdot \sigma(t) \cdot V_\sigma(t, X(t), \sigma(t)) \cdot 1\mathrm{bp},\\
&\mathrm{IM}_{\mathrm{Curvature}}(t) = \min\left\{1, \frac{14\mathrm{Days}}{\text{Time to maturity}}\right\} \cdot \sigma(t) \cdot V_\sigma(t, X(t), \sigma(t)) \cdot 1\mathrm{bp}.
\end{aligned}$$

ISDA SIMM was developed in response to the financial crisis, ISDA SIMM attempts to incorporate the impact indirectly. ISDA SIMM adopts the conservative method, called 1+3 standard, to estimate SIMM coefficients corresponding to the volatility of the risk factors. These coefficients are estimated by historical data that consists of one stress year [4] plus three non-stress years. Thus, IM is conservatively calibrated upward assuming a certain jump. However, it does not accurately model the uncertainty of the impact. [5]

2.3 Impact on financial markets due to the SIC default

In this section, we model the financial market and impact of a certain SIC default.

First, we adopt that the processes X and σ for $t \neq \tau$ are governed by a generalized stochastic volatility model.

$$\begin{aligned}
dX(t) &= \mu_X(t, X(t), \sigma(t))dt + \sigma_X(t, X(t), \sigma(t))dW_X(t),\\
d\sigma(t) &= \mu_\sigma(t, X(t), \sigma(t))dt + \sigma_\sigma(t, X(t), \sigma(t))dW_\sigma(t).
\end{aligned}$$

Here, $\mu_X, \mu_\sigma, \sigma_X, \sigma_\sigma$ are the drift and volatilities for the underlying asset and its volatility, respectively, and W_X, W_σ are assumed to be standard Brownian motions under a physical probability measure $\mathbf{P}$ with correlation $\rho \in [-1, 1]$.

In this study, similar to the prior research and ISDA SIMM, we introduce the impact of the SIC default as a jump in the risk factors that constitute the value of derivative price. We denote the jump in X and σ as a random variable $J = (J_X, J_\sigma)$.

[3] We focus on the general counterparty risk. Therefore we disregard IM related to Base Correlation specific to credit risk and related to concentration risk.

[4] The data is selected to maximize volatility among all historical data. At the time of writing, data including the default of LB may be used.

[5] In addition, ISDA SIMM requires financial institutions to conduct a 1+3 back test. This involves conducting a back test using historical data on the current position. Here too, historical data that includes the most recent three years plus one year of shock periods[4] is used to intentionally increase volatility.

We assume that the filtration $\mathcal{F}$ and J are independent.

$$X(\tau) = X(\tau-) + J_X, \quad \sigma(\tau) = \sigma(\tau-) + J_\sigma.$$

Then, we can decompose the equation (1) into the jump due to the default and the change in value following the jump.

$$\mathrm{PE}(t) = \Big((V(\tau + \delta^*) - V(\tau) - \mathrm{IM}(\tau-)) + (V(\tau) - V(\tau-)) \Big)^+ \Big|_{\tau = t}. \tag{2}$$

From Ito's formula, it follows that if the function V is a $C^{1,2,2}$-function, we can express the value process of V as,

$$\begin{aligned}
dV(t) &= dV(t, X(t), \sigma(t)) \\
&= V_t(t)dt + V_x(t)dX(t) + V_\sigma(t)d\sigma(t) + \frac{1}{2}V_{xx}(t)d\langle X\rangle(t) + \frac{1}{2}V_{\sigma\sigma}(t)d\langle\sigma\rangle(t) + V_{x\sigma}(t)d\langle X\sigma\rangle(t) \\
&= V_x(t)\sigma_X(t, X(t), \sigma(t))dW_X(t) + V_\sigma(t)\sigma_\sigma(t, X(t), \sigma(t))dW_\sigma(t) \\
&\quad + \Big[V_t(t) + V_x(t)\mu_X(t, X(t), \sigma(t)) + \frac{1}{2}V_{xx}(t)\sigma_X^2(t, X(t), \sigma(t)) \\
&\quad + V_\sigma(t)\mu_\sigma(t, X(t), \sigma(t)) + \frac{1}{2}V_{\sigma\sigma}(t)\sigma_\sigma^2(t, X(t), \sigma(t)) \\
&\quad + \rho V_{x\sigma}(t)\sigma_X(t, X(t), \sigma(t))\sigma_\sigma(t, X(t), \sigma(t)) \Big] dt.
\end{aligned} \tag{3}$$

Given that MPoR δ^* is a sufficiently short period [6] , we can apply a discrete approximation to equation (3) using random variables Z_X and Z_σ that follow standard normal distribution with correlation $\rho \in [-1, 1]$.

$$W_X(t + \delta^*) - W_X(t) \simeq A_D(t)Z_X, \quad W_\sigma(t + \delta^*) - W_\sigma(t) \simeq A_V(t)Z_\sigma,$$

Then the first term of equation (2) can be expressed as follows.

$$\begin{aligned}
V(t + \delta^*) - V(t) &\simeq A_D(t)Z_X + A_V(t)Z_\sigma + A_\delta(t), \\
A_D(t) &= A_D(t-, J) = V_x(t-, J)\sigma_X(t-, J)\sqrt{\delta^*}, \\
A_V(t) &= A_V(t-, J) = V_\sigma(t-, J)\sigma_\sigma(t-, J)\sqrt{\delta^*}, \\
A_\delta(t) &= A_\delta(t-, J) \\
&= \Big(V_t(t-, J) + V_x(t-, J)\mu_X(t-, J) + \frac{1}{2}V_{xx}(t-, J)\sigma_X^2(t-, J) \\
&\quad + V_\sigma(t-, J)\mu_\sigma(t-, J) + \frac{1}{2}V_{\sigma\sigma}(t-, J)\sigma_\sigma^2(t-, J)
\end{aligned}$$

[6] 10 business days is used in risk management according to Basel Committee on Banking Supervision (BCBS) (2016). In the numerical experiments conducted later, we use 14 days ($\delta^* = 14\mathrm{Days}/365\mathrm{Days}$).

$$+ \rho V_{x\sigma}(t-, J)\sigma_X(t-, J)\sigma_\sigma(t-, J)\Big)\delta^*.$$

We label the jump in value due to the SIC default, the second term of equation (2), as,

$$A_J(t) = A_J(t-, J) = V(t) - V(t-) = V(t-, J) - V(t-),$$

If we set $A_C(t) = A_\delta(t) - \mathrm{IM}(t-)$, equation (2) becomes,

$$\mathrm{PE}(t) = \left(A_D(t)Z_X + A_V(t)Z_\sigma + A_C(t) + A_J(t)\right)^+ . \quad (4)$$

Therefore, we can see that PE is calculated by the partial derivatives of V.

In the next section, we will define CPR indicators using PE and derive their approximate formulas.

3 Analytical Approach to CPR for SICs

In this section, we introduce the general CPR indicators and derive the approximate formulas under a jump in risk factors. Furthermore, to analyze the risk in greater detail, we decompose PFE, one of the indicators, into Factor PFE of Delta, Vega, Curvature and Jump components.

3.1 CPR Indicators and Approximate formulas

We focus on Mratio, PFE, and EPE as representative CPR indicators.

Kitani and Nakagawa (2024) define Margin Conservation Ratio (Mratio) as the probability that PE equals zero.

$$\mathrm{Mratio}(t) = \mathbf{P}\left(\mathrm{PE}(t) = 0 | \mathcal{F}_{t-}\right). \quad (5)$$

where Φ is the cumulative distribution function of the standard normal distribution, and we also define ϕ as its probability density function, which will be used in later sections.

Mratio serves to verify whether regulatory requirements are satisfied or not. Regulations require IM to cover 99 % of the loss, so if Mratio is 99 % or higher, the regulatory requirements are satisfied. Conversely, if Mratio falls below 99 %, the regulatory requirements are not satisfied. We then quantify the amount of risk.

Gregory (2020) defines Potential Future Exposure (PFE) as the 99th percentile of losses and Expected Positive Exposure (EPE) as the expectation of PE. PFE is conceptually similar to Value at Risk (VaR).

$$\begin{aligned} \mathrm{PFE}(t) &= \operatorname{ess.inf}\left\{y \in \mathbb{R} \,\middle|\, \mathbf{P}\left(\mathrm{PE}(t) \geq y \mid \mathcal{F}_{t-}\right) \leq 0.01\right\}, \\ \mathrm{EPE}(t) &= \mathbf{E}\left[\mathrm{PE}(t) \mid \mathcal{F}_{t-}\right]. \end{aligned} \quad (6)$$

PFE and EPE are calculated in monetary terms. In this study we measure the amount of the loss by PFE, which is highly compatible with VaR, is used in risk management. Although EPE is not employed in our numerical experiments, it is widely used in CVA calculations. In the numerical experiments conducted later, we first examine whether IM satisfied regulatory requirements or not by checking Mratio. If Mratio is below 99%, we check PFE to measure the amount of the loss.

In the case of No-Jump, Kitani and Nakagawa (2024) simplify PE employing the random variable Z that follows a standard normal distribution.

$$
\begin{aligned}
\mathrm{PE}(t) &\simeq A_Z(t)Z + A_M(t), \\
A_Z(t) &= \sqrt{A_D^2(t) + 2\rho A_D(t)A_V(t) + A_V^2(t)}, \\
A_M(t) &= A_C(t) + A_J(t), \\
Z &= \frac{A_D(t)Z_X + A_V(t)Z_\sigma}{A_Z(t)}.
\end{aligned}
\tag{7}
$$

Then, since A_Z and A_M are $\mathcal{F}_t$-measurable functions, we can derive the approximate formulas for the CPR indicators as follows.

$$
\begin{aligned}
\mathrm{Mratio}(t) &\simeq \Phi\left(-\frac{A_M(t)}{A_Z(t)} \,\middle|\, \mathcal{F}_{t-}\right), \\
\mathrm{PFE}(t) &\simeq \operatorname{ess.inf}\left\{ y \in \mathbb{R} \,\middle|\, \mathbf{P}\left((A_Z(t)Z + A_M(t))^+ \geq y \,\middle|\, \mathcal{F}_{t-}\right) \leq 0.01 \right\}, \\
\mathrm{EPE}(t) &\simeq \mathbf{E}\left[(A_Z(t)Z + A_M(t))^+ \,\middle|\, \mathcal{F}_{t-}\right].
\end{aligned}
$$

Next, we consider the jump $J = (J_X, J_\sigma)$. Ideally, the jump distribution should be defined as a parametric function. However, since there are few examples of SICs defaults, it is difficult to estimate the parameters. In reality, stress scenarios under a certain assumption are more appropriate. Therefore, in this study, we assume J as a random variable with discrete probability distribution.

$$
n = 1, 2 \cdots N, \quad j_n = (j_{n,X}, j_{n,\sigma}), \quad p_n = \mathbf{P}(J = j_n).
$$

If we set $A_{n,Z}(t) = A_Z(t-, j_n)$, $\quad A_{n,M}(t) = A_M(t-, j_n)$, $A_{n,Z}$ and $A_{n,M}$ are $\mathcal{F}_{t-}$-measurable functions. Since $\mathcal{F}$ and J are independent, applying Kitani and Nakagawa (2024) equations (5) and (6) are equivalent to,

$$
\begin{aligned}
\mathrm{Mratio}(t) &= \sum_{n=1}^{N} \Phi\left(-\frac{A_{n,M}(t)}{A_{n,Z}(t)}\right) p_n(t), \\
\mathrm{EPE}(t) &= \sum_{n=1}^{N} \left[\frac{A_{n,Z}(t)}{\sqrt{2\pi}} e^{-\frac{A_{n,M}^2(t)}{2A_{n,Z}^2(t)}} + A_{n,M}(t)\Phi\left(\frac{A_{n,M}(t)}{A_{n,Z}(t)}\right) \right] p_n(t),
\end{aligned}
$$

For PFE, let $\mathrm{Mratio}'(t,y) = \mathbf{P}\left((A_Z(t)Z - A_M(t) - y)^+ = 0 \;\middle|\; \mathcal{F}_{t-} \right)$, then,

$$
\begin{aligned}
\mathrm{PFE}(t) &= \operatorname{ess.inf}\left\{ y \in \mathbb{R} \;\middle|\; \mathbf{P}\left((A_{n,Z}(t)Z + A_M(t-))^+ \geq y \;\middle|\; \mathcal{F}_{t-} \right) \leq 0.01 \right\}, \\
&= \operatorname{ess.inf}\left\{ y \in \mathbb{R} \;\middle|\; 1 - \mathrm{Mratio}'(t,y) \leq 0.01 \right\}, \\
&= \operatorname{ess.inf}\left\{ y \in \mathbb{R} \;\middle|\; \mathrm{Mratio}'(t,y) \geq 0.99 \right\}.
\end{aligned}
$$

Therefore, PFE can be evaluated numerically. [7]

Thus the CPR indicators can be computed once J and the partial derivatives of V are specified.

3.2 Factor decomposition of exposure

Furthermore, to analyze the CPR we decompose PFE into four factors: Delta (D), Vega (V), and Curvature (C), which are the basic factors for calculating IM in ISDA SIMM, and Jump (J).

First, PE derived in equation (4) can be decomposed into four factor PFEs.

$$
\mathrm{PE}(t) = (\mathrm{PE}_D(t-) + \mathrm{PE}_V(t-) + \mathrm{PE}_C(t-) + \mathrm{PE}_J(t-))^+,
$$

where,

$$
\begin{aligned}
\mathrm{PE}_D(t) &= A_D(t)Z_X + V_x(t)\mu_X(t)\delta^* - \mathrm{IM}_{Delta}(t-), \\
\mathrm{PE}_V(t) &= A_V(t)Z_\sigma + V_\sigma(t)\mu_\sigma(t)\delta^* - \mathrm{IM}_{Vega}(t-), \\
\mathrm{PE}_C(t) &= A_C(t) - (V_x(t)\mu_X(t) + V_\sigma(t)\mu_\sigma(t))\,\delta^* + \mathrm{IM}_{Delta}(t-) + \mathrm{IM}_{Vega}(t-), \\
\mathrm{PE}_J(t) &= A_J(t).
\end{aligned}
$$

Then, based on this decomposition, we introduce four factor PFEs.

For explicit calculations, similar to equation (7), we aggregate Z_X and Z_σ into Z and define PE_Z.

$$
\begin{aligned}
\mathrm{PE}_Z(t) &= \mathrm{PE}_D(t) + \mathrm{PE}_V(t) \\
&= A_Z(t)Z + (V_x(t)\mu_X(t) + V_\sigma(t)\mu_\sigma(t))\,\delta^* - (\mathrm{IM}_{Delta}(t-) + \mathrm{IM}_{Vega}(t-)).
\end{aligned}
$$

First, we define the factor PFEs with the following conditional expectation.

$$
f \in \{D, V, Z, C, J\}, \quad \mathrm{PFE}_f(t) = \mathbf{E}\left[\mathrm{PE}_f(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t) \right].
$$

[7] Since $A_{n,Z}(t) > 0$ from equation (7) Mratio' is monotonically decreasing in y, and therefore PFE can be uniquely calculated numerically.

$$
\frac{\partial \mathrm{Mratio}'}{\partial y}(t,y) = -\sum_{n=1}^{N} \phi\left(\frac{A_{n,M}(t-) - y}{A_{n,Z}(t)} \right) \frac{p_n(t)}{A_{n,Z}(t)} \leq 0
$$

We can see that the sum of PFE_D and PFE_V equals PFE_Z under this definition.

$$\begin{aligned}
\mathrm{PFE}_D(t) + \mathrm{PFE}_V(t) &= \mathbf{E}\left[\mathrm{PE}_D(t) + \mathrm{PE}_V(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right] \\
&= \mathbf{E}\left[\mathrm{PE}_Z(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right] = \mathrm{PFE}_Z(t).
\end{aligned}$$

We can confirm that under this definition the aggregation of all factor PFEs reproduces the original PFE.

$$\begin{aligned}
&\mathrm{PFE}(t) = \mathrm{PFE}_D(t) + \mathrm{PFE}_V(t) + \mathrm{PFE}_J(t) + \mathrm{PFE}_C(t), \\
&\mathrm{PFE}_Z(t) = \mathrm{PFE}_D(t) + \mathrm{PFE}_V(t).
\end{aligned}$$

We set J as a discrete random variable with a set of N elements, $\{j_n \mid n = 1, \cdots, N\}$. Then, we define factor $\mathrm{PE}_{n,f}$ and $\mathrm{PFE}_{n,f}$ in a form further conditioned by J.

$$\begin{aligned}
&f \in \{D, V, Z, C, J\}, \quad n = 1, \cdots, N, \\
&\mathrm{PE}_{n,f}(t) = \mathrm{PE}_f(t) \big|_{J=j_n} = \mathrm{PE}_f(t-, j_n), \\
&\mathrm{PFE}_{n,f}(t) = \mathbf{E}\left[\mathrm{PE}_{n,f}(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right].
\end{aligned}$$

Since J are disjoint sets, from Bayes' formula, factor PFEs can be calculated as follows,

$$\begin{aligned}
\mathrm{PFE}_f(t) &= \sum_{n=1}^{N} \mathbf{E}\left[\mathrm{PE}_{n,f}(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right] \mathbf{P}\left(J = j_n \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}(t)\right) \\
&= \sum_{n=1}^{N} \mathrm{PFE}_{n,f}(t) \frac{\mathbf{P}\left(J = j_n \cap \mathrm{PE}(t) = \mathrm{PFE}(t)\right)}{\mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t)\right)} \\
&= \frac{\displaystyle\sum_{n=1}^{N} \mathrm{PFE}_{n,f}(t) \cdot \mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t) \;\middle|\; J = j_n\right) p_n(t)}{\displaystyle\sum_{n=1}^{N} \mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t) \;\middle|\; J = j_n\right) p_n(t)}.
\end{aligned}$$

Here, since $\mathrm{PE}_{n,C}$ and $\mathrm{PE}_{n,J}$ are functions that do not contain random variables, both conditional factor PFEs can be treated as constants.

$$\mathrm{PFE}_{n,C}(t) = \mathrm{PE}_{n,C}(t), \quad \mathrm{PFE}_{n,J}(t) = \mathrm{PE}_{n,J}(t).$$

Then, $\mathrm{PFE}_{n,Z}$ can be obtained as follows.

$$\begin{aligned}
\mathrm{PFE}_{n,Z}(t) &= \mathrm{PFE}(t) - \left(\mathrm{PFE}_{n,C}(t) + \mathrm{PFE}_{n,J}(t)\right) \\
&= \mathrm{PFE}(t) - \left(\mathrm{PE}_{n,C}(t) + \mathrm{PE}_{n,J}(t)\right).
\end{aligned}$$

Furthermore, since $\mathrm{PE}_{n,Z}$ is a random variable that follows a normal distribution, the conditional probability can be calculated as follows.

$$\begin{aligned}
\mathbf{P}\left(\mathrm{PE}(t) = \mathrm{PFE}(t) \;\middle|\; J = j_n\right) &= \mathbf{P}\left(\mathrm{PE}_{n,Z}(t) = \mathrm{PFE}_{n,Z}(t)\right) \\
&= \mathbf{P}\left(Z = \frac{\mathrm{PFE}_{n,Z}(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]}{A_Z(t-)}\right) \\
&= \phi\left(\frac{\mathrm{PFE}_{n,Z}(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]}{A_Z(t-)}\right). \\
\mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right] &= \left(V_x(t)\mu_X(t) + V_\sigma(t)\mu_\sigma(t)\right)\delta^* - \left(\mathrm{IM}_{Delta}(t-) + \mathrm{IM}_{Vega}(t-)\right).
\end{aligned}$$

Since PE_D and PE_V are random variables that follow a normal distribution, the conditional expectation $\mathrm{PFE}_{n,D}$ can be calculated employing their correlation ρ_Z between PE_D and PE_V as follows. [8]

$$\begin{aligned}
\mathrm{PFE}_{n,D}(t) &= \mathbf{E}\left[\mathrm{PE}_{n,D}(t) \;\middle|\; \mathrm{PE}(t) = \mathrm{PFE}\right] \\
&= \mathbf{E}\left[\mathrm{PE}_{n,D}(t) \;\middle|\; \mathrm{PE}_{n,Z}(t) = \mathrm{PFE}_Z(t)\right] \\
&= \mathbf{E}\left[\mathrm{PE}_{n,D}(t)\right] \\
&\quad + \frac{A_D^2(t) + \rho_Z A_D(t) A_V(t)}{A_Z^2(t)}\left(\mathrm{PFE}_Z(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]\right), \qquad (8) \\
\rho_Z &= \frac{A_Z^2(t) - A_D^2(t) - A_V^2(t)}{2A_D(t)A_V(t)}. \\
\mathbf{E}\left[\mathrm{PE}_{n,D}(t)\right] &= V_x(t)\mu_X(t)\delta^* - \mathrm{IM}_{Delta}(t-).
\end{aligned}$$

Similarly, we can calculate PFE_V as follows.

$$\begin{aligned}
\mathrm{PFE}_{n,V}(t) &= \mathbf{E}\left[\mathrm{PE}_{n,V}(t)\right] + \frac{A_V^2(t) + \rho_Z A_D(t) A_V(t)}{A_Z^2(t)}\left(\mathrm{PFE}_Z(t) - \mathbf{E}\left[\mathrm{PE}_{n,Z}(t)\right]\right). \\
\mathbf{E}\left[\mathrm{PE}_{n,V}(t)\right] &= V_\sigma(t)\mu_\sigma(t)\delta^* - \mathrm{IM}_{Vega}(t-).
\end{aligned}$$

4 Numerical Experiments for swaptions under the SABR model

In this section we apply the CPR indicators in numerical experiments on a European swaption under the SABR model. We then conduct numerical experiments with several jump scenarios. As a result, we can confirm that IM calculated by ISDA SIMM may not satisfy the regulatory requirements.

[8] See the appendix for detailed derivation.

4.1 Pricing swaptions under the SABR model

In the numerical experiments we evaluate a European swaption under the SABR model as a case study.

As mentioned above, regulations require derivatives transactions to be centrally cleared. However, transactions with non-linear risks such as options are not subject to the regulations and are traded in OTC because it is difficult for central counterparties to specify a concrete calculation method of margins, similar to regulations in OTC derivatives. European Swaptions are commonly traded as a plain vanilla interest rate derivatives and are not centrally cleared due to its non-linear risk.

In this option the underlying asset is the forward swap rate X and when the option is exercised at the maturity, a swap transaction can be initiated [9] at strike rate K. For simplicity, we normalize the current annuity value to unity. The payoffs of options at maturity T with the strike rate K are as follows:

$$V^C(T) = (X(T) - K)^+, \quad V^P(T) = (K - X(T))^+$$

Next, we set the valuation model as the SABR model, a special type of stochastic volatility models proposed by Hagan et al. (2002). According to Brigo and Mercurio (2001) and Andersen and Piterbarg (2010), this model is commonly used in the interest rate derivatives market to represent the skew of the options market by setting the underlying asset as the forward swap rate.

In this model, the forward swap rate and its volatility satisfy the following stochastic differential equation under the swap measure $\mathbf{Q}$.

$$\begin{aligned} dX(t) &= \sigma(t)X^{\beta}(t)dW_X^{\mathbf{Q}}(t), \\ d\sigma(t) &= \nu\sigma(t)dW_\sigma^{\mathbf{Q}}(t), \end{aligned}$$

Here, $W_X^{\mathbf{Q}}$ and $W_\sigma^{\mathbf{Q}}$ are standard Brownian motions with $\rho \in [-1, 1]$ under the measure $\mathbf{Q}$. Furthermore, each parameter is a constant such that $\sigma(0) > 0, \beta \in [0, 1]$ and $\nu > 0$.

The CPR indicators such as Mratio or PFE are originally calculated under $\mathbf{P}$. Therefore, it is necessary to transform the measure from $\mathbf{Q}$ to $\mathbf{P}$.

Here, we assume that $\theta_X(t)$ and $\theta_\sigma(t)$ are given as $(\mathcal{F}_t)$-adapted processes satisfying the following equations.

$$dW_X^{\mathbf{Q}}(t) = dW_X(t) - \theta_X(t)dt, \quad dW_\sigma^{\mathbf{Q}}(t) = dW_\sigma(t) - \theta_\sigma(t)dt.$$

Then, according to Girsanov-Maruyama theorem, we can transform the Brownian motions from $dW_X^{\mathbf{Q}}(t), dW_\sigma^{\mathbf{Q}}(t)$ under $\mathbf{Q}$ to $dW_X(t), dW_\sigma(t)$ under $\mathbf{P}$. In other words, the dynamics of X and σ under $\mathbf{P}$ can be determined.

In the numerical experiments conducted later, we set the drift term under $\mathbf{P}$ in the same way as Kitani and Nakagawa (2024).

$$\mu_X(t) = \theta_X(t)\sigma(t)X^{\beta}(t), \quad \mu_\sigma(t) = \theta_\sigma(t)\nu\sigma(t).$$

[9] Depending on the contract, some options are cleared by exchanging cash.

From Hagan et al. (2002) there are two types of approximation formulas for European option price under the SABR model: a Log-Normal type and a Normal type.

The first one, which is closer to the log normal model, is following;

$$
\begin{aligned}
V^{C(L)}(T-t, X(t), \sigma_L(t), K) &\simeq X(t)\Phi\left(d_+(t)\right) - K\Phi\left(d_-(t)\right), \\
V^{P(L)}(T-t, X(t), \sigma_L(t), K) &\simeq -X(t)\Phi\left(-d_+(t)\right) + K\Phi\left(-d_-(t)\right).
\end{aligned}
$$

where,

$$
\begin{aligned}
z_L(t) &:= z_L(X(t), \sigma_L(t), \beta, \nu, K) = \frac{(X(t)K)^{\frac{1-\beta}{2}} \ln \frac{X(t)}{K}}{\sigma(t)}\nu, \\
\mathfrak{X}(z, \rho) &:= \ln\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right), \\
\Theta_L(t) &:= \Theta_L(X(t), \sigma(t), \beta, \rho, \nu, K) = \frac{1}{24}\frac{\sigma^2(t)\left(1-\beta\right)^2}{\left(X(t)K\right)^{1-\beta}} + \frac{\rho\nu\sigma(t)\beta}{4\left(X(t)K\right)^{\frac{1-\beta}{2}}} + \frac{2-3\rho^2}{24}\nu^2,
\end{aligned}
$$

$$
\begin{aligned}
\sigma_L(t) &:= \sigma_L(T-t, X(t), \sigma(t), \beta, \rho, \nu, K) \\
&= \frac{\sigma(t)}{(X(t)K)^{\frac{1-\beta}{2}}\left[1+\frac{(1-\beta)^2}{24}\left(\ln\frac{X(t)}{K}\right)^2+\frac{(1-\beta)^4}{1920}\left(\ln\frac{X(t)}{K}\right)^4\right]}\frac{z_L(t)}{\mathfrak{X}(z_L(t), \rho)}\left[1+\Theta_L(t)\left(T-t\right)\right],
\end{aligned}
$$

$$
d_+(t) := d_+(T-t, X(t), \sigma_L(t), K) = \frac{\ln\frac{X(t)}{K} + \frac{\sigma_L^2(t)(T-t)}{2}}{\sigma_L(t)\sqrt{T-t}}, \quad d_-(t) := d_+(t) - \sigma_L(t)\sqrt{T-t}.
$$

The other, which is closer to the normal model, is following;

$$
\begin{aligned}
V^{C(N)}(T-t, X(t), \sigma_N(t), K) &\simeq (X(t)-K)\Phi(d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)), \\
V^{P(N)}(T-t, X(t), \sigma_N(t), K) &\simeq (K-X(t))\Phi(-d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)),
\end{aligned}
$$

where,

$$
\begin{aligned}
z_N(t) &:= z_N(X(t), \sigma(t), \beta, \nu, K) = \frac{\nu}{\sigma(t)}\frac{X(t)^{1-\beta}-K^{1-\beta}}{1-\beta}, \\
\mathfrak{X}(z, \rho) &:= \ln\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right), \\
\Theta_N(t) &:= \Theta_N(X(t), \sigma(t), \beta, \rho, \nu, K) \\
&= \frac{1}{24}\frac{\sigma^2(t)\beta(\beta-2)(1-\beta)^2\left(\ln\frac{X(t)}{K}\right)^2}{(X^{1-\beta}(t)-K^{1-\beta})^2} + \frac{\rho\nu\sigma(t)}{4}\frac{X^\beta(t)-K^\beta}{X(t)-K} + \frac{2-3\rho^2}{24}\nu^2,
\end{aligned}
$$

$$
\begin{aligned}
\sigma_N(t) &:= \sigma_N(T-t, X(t), \sigma(t), \beta, \rho, \nu, K) \\
&= \frac{\sigma(t)(1-\beta)(X(t)-K)}{X^{1-\beta}(t)-K^{1-\beta}} \frac{z_N(t)}{\mathfrak{X}(z_N(t),\rho)} \left[1+\Theta_N(t)(T-t)\right], \\
d(t) &:= d(T-t, X(t), \sigma_N(t), K) = \frac{X(t)-K}{\sigma_N(t)\sqrt{T-t}}.
\end{aligned}
$$

From these approximation formulas we can derive the partial derivatives of V ($V_x, V_\sigma, V_{xx}, V_{\sigma\sigma}$, and $V_{x\sigma}$). [10] As a result, we can calculate the CPR indicators specifically.

4.2 Assumptions for Numerical Experiments

We assume that the swaption strike price is 3% or 5% (OTM) and its option term is $T-t=1$ Year. [11] The payoff at maturity T are as follows;

$$
V^C(T) = (X(T)-5\%)^+, \quad V^P(T) = (3\% - X(T))^+.
$$

We set the coefficient of the SABR model. First, we suppose $\beta = 0.75$ because the interest rate level is high and the underlying asset is close to a lognormal process with $\beta = 1$. In addition, $\nu = 20\%$ for the volatility of the volatility and $\rho = 0.5$ for the correlation between the underlying asset and its volatility.

$$
\begin{aligned}
&dX(t) = \sigma(t)X^{0.75}(t)dW_X(t), \\
&d\sigma(t) = 0.20\sigma(t)dW_\sigma(t), \\
&dW_X(t)dW_\sigma(t) = 0.50dt.
\end{aligned}
$$

The MPoR δ^* is set to 14 days based on Basel Committee on Banking Supervision (BCBS) (2016).

$$
\delta^* = \frac{10\text{Business Days}}{1\text{Year}} \simeq \frac{14\text{Days}}{365\text{Days}}.
$$

We assume the drift $\mu_X = 0$ and $\mu_\sigma = 0$ based on Kitani and Nakagawa (2024). These parameter values are chosen to reflect typical market conditions for interest rate swaptions.

In practice, the SIMM coefficients (RW, VRW) have different values at different maturity [12] , but for the sake of simplicity, we assume RW= 60 and VRW= 0.20 for all maturity, according to the SIMM coefficients in the regular currencies as shown in Table 1.

[10] For specific derivations, refer to the Appendix.

[11] In the market convention the strike are generally used ATM, but since the underlying asset price changes after the trade, the strike of almost all transactions are not ATM. In addition, since active funds, which frequently use options trading, mainly trade OTM, which are low option price and high return, we mainly observe OTM options in this study.

[12] ISDA SIMM defines three types of RW for each currency (Regular, Low volatility, High volatility). Regular covers USD, EUR, GBP, CHF, AUD, NZD, CAD, SEK, NOK, DKK, HKD, KRW, SGD and TWD. Low volatiloty is only for the JPY, while other currencies are classified as high volatility. VRW is common to all currencies.

Table 1: The SIMM coefficients version 2.7 revised in Dec 2024

type	ccy	2w	1m	3m	6m	1yr	2yr	3yr	5yr	10yr	15yr	20yr	30yr
RW	Regular	109	106	91	69	68	68	66	61	59	59	57	65
	Low-vol	15	21	10	10	11	15	18	23	25	23	23	25
	High-vol	171	102	94	96	105	96	99	93	99	100	101	96
VRW	All	0.20											

We consider four jump scenarios below. The expectation of both J_X and J_σ is identical across all scenarios excluding No-jump scenario.

- Stochastic
  This scenario is designed to resemble the 1+3 standard employed in ISDA SIMM. The method estimates the SIMM coefficients by histrical data that consists of one stress year, such as the LB cases, plus three non-stress years.
  In this scenario, with 25 % probability underlying asset and its volatility jump like the LB cases with a decline in interest rate minus 1 % and a spike in its implied volatility plus 20 %. With 75 % probability there is no jump.

$$j_1 = (-1.0\%, +20\%), \quad p_1 = 25.0\%,$$
$$j_2 = (\pm 0.0\%, \pm 0\%), \quad p_2 = 75.0\%.$$

- Constant
  This scenario resembles to estimation of the SIMM coefficients. In this scenarios the jump is not probabilistic and equals the expectation of jump in Stochastic scenario. [13]

$$j_1 = (-0.25\%, +5\%), \quad p_1 = 100\%.$$

- Two-Steps
  As an extreme jump example, we decompose j_1 in Stochastic scenario into j_1 and j_2.

$$j_1 = (-1.5\%, +25\%), \quad p_1 = 12.5\%,$$
$$j_2 = (-0.5\%, +15\%), \quad p_2 = 12.5\%,$$
$$j_3 = (\pm 0.0\%, \pm 0.0\%), \quad p_3 = 75.0\%.$$

- No-Jump
  There is no jump.

$$j_1 = (\pm 0.0\%, \pm 0.0\%), \quad p_1 = 100\%.$$

The scenarios and parameters assumed in the numerical experiments in this study are shown in Tables 2 and 3.

[13] ISDA estimate SIMM coefficients as the 99% value of variation in 14 days calculated in four-years data. Therefore, SIMM coefficients may be more conservative than the Constant scenario calculated as an expectation.

Table 2: Scenarios of Jump

Scenario	$J = (J_X, J_\sigma)$	p_n	note
Two-Steps	$(-1.5\%, +25\%)$	12.5%	Decomposition of a stochastic
	$(-0.5\%, +15\%)$	12.5%	into two parts
	$(\pm0.0\%, \pm0.0\%)$	75.0%	
Stochastic	$(-1.0\%, +20\%)$	25.0%	based on 1+3 standard
	$(\pm0.0\%, \pm0.0\%)$	75.0%	
Constant	$(-0.25\%, +5\%)$	100%	based on the calculations of SIMM coefficients
No-Jump	$(\pm0.0\%, \pm0.0\%)$	100%	Nothing

Table 3: Parameters

transaction	K	3%,5% (OTM)
	$T - t$	1.0 Year
pricing model	β	0.75
	ν	20 %
	ρ	0.50
	μ_X, μ_σ	0.00
	δ^*	14 / 365
ISDA SIMM	RW	60
	VRW	0.20

4.3 Result of Numerical Experiments

We first verify whether IM satisfies the regulatory requirements or not by evaluating Mratio. If it is 99% or higher, we can confirm that the regulatory requirements are satisfied. But, if not, they are not satisfied and we measure the risk amount by PFE.

In the numerical experiments, we conduct Mratio and PFE for the volatility σ in the range of 7.5% to 22.5%. The same analysis was conducted for the relationship of the underlying asset X horizontally from 1.75% to 6.25%. For these values, X is the central level since the 1990s. Although we cannot observe σ directly, it is roughly considered the central level.

Fig. 4 and 5 show that in No-Jump scenario, since Mratio ,depicted by the blue line, stays near the 99% and this means that IM satisfy the regulatory requirements. Conversely, in the other three scenarios, Mratio often falls below 99%, and this indicates IM may not satisfy the regulatory requirements even in low volatility.

Next, we evaluate PFE in these three scenarios. It is observed that PFE in Two-Steps and Stochastic scenarios are larger than those in Constant scenarios. This result stems from the fact that in these two scenarios the impact is probabilistic.

Furthermore, PFE remained constant regardless of the level of σ.

On the other hand, Fig. 6 and 7 show that PFE increases as the underlying asset price approaches At The Money (ATM, $X = K$).

Subsequently, we compare the differences in PFE between call and put options. In this study we assume the impact is the Lehman Brothers case with a decline in interest rates and a spike in their implied volatility. Therefore, in Put option as both

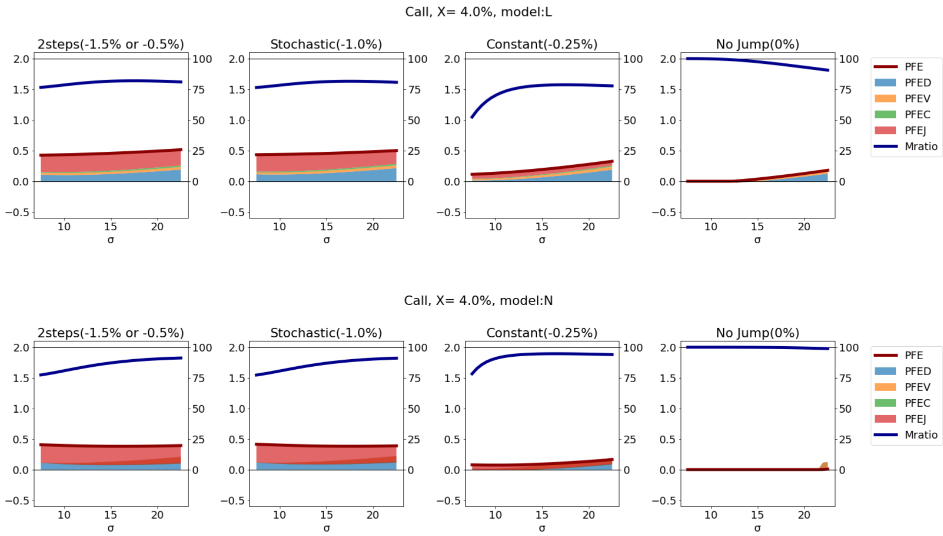

Fig. 4: By scenario of jump, the volatility and CPR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

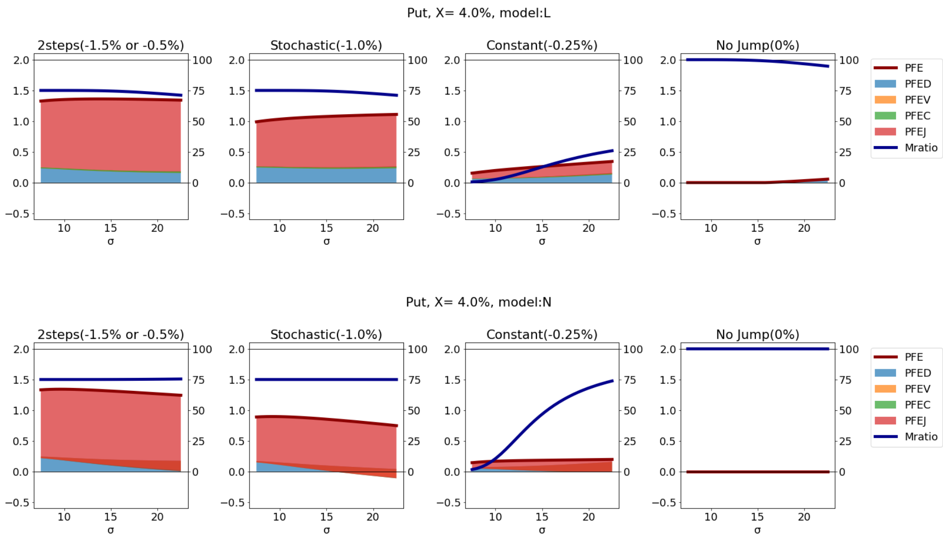

Fig. 5: By scenario of jump, the volatility and CPR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

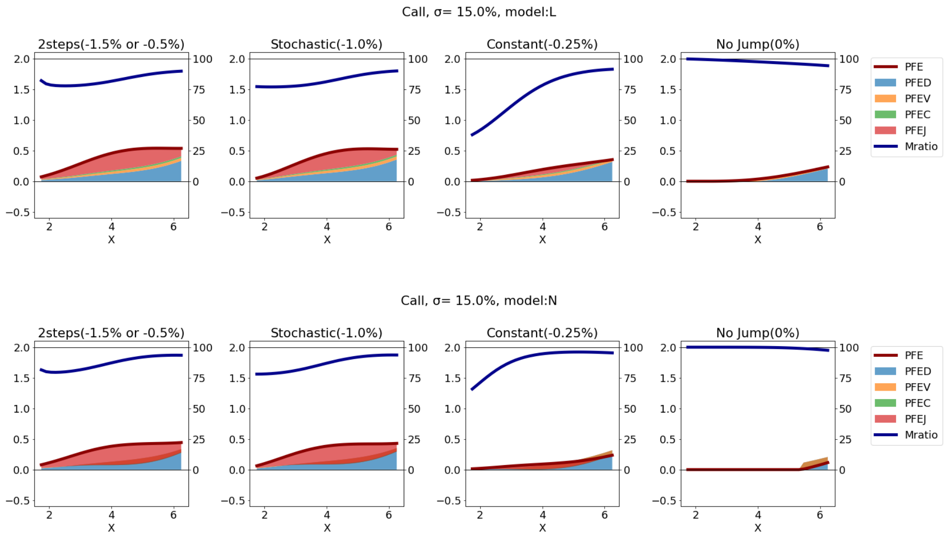

Fig. 6: By scenario of jump, the underlying asset and CPR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

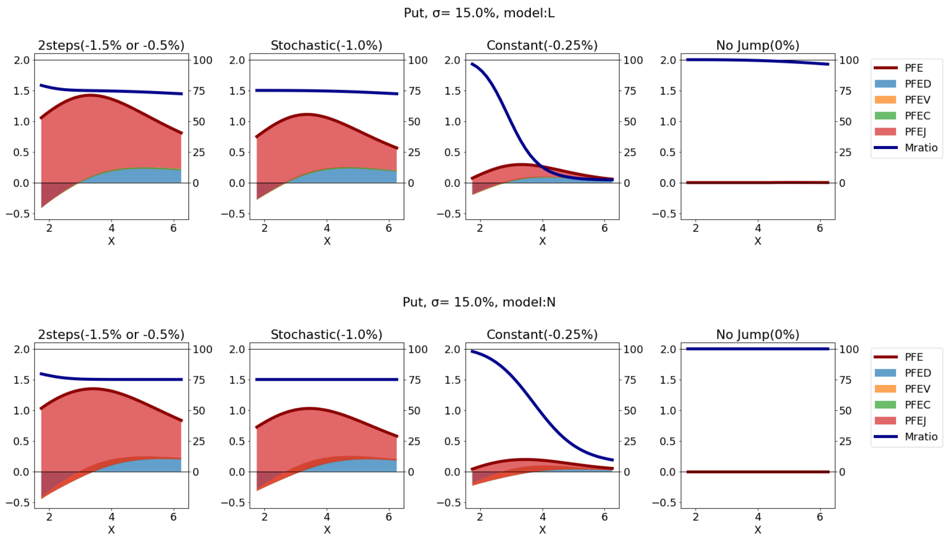

Fig. 7: By scenario of jump, the volatility and CPR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

the change contribute to increase exposure, you can see that PFE of Put option are larger than those of Call option.

Finally, we check the factor PFE. It is evident that the jump factor PFE, depicted by the red area, is the largest. In particular, in Two-Steps and Stochastic scenarios where the impact is probabilistic the factor PFE of jump is larger than that in the other scenarios. The results confirm that jump is a significant driver in PFE calculation.

We conduct numerical experiments with jump to assess whether IM calculated by ISDA SIMM cover the exposure or not even under the impact of the SIC default. As a result, Mratio clearly falls below 99 % due to the jump.

This result suggest that depending on the transaction type and the nature of the jump under the impact IM calculated by ISDA SIMM may not eliminate CPR to the regulatory level. Furthermore, it is crucial to accurately model the impact for the measurement and management of the CPR.

We next examine whether MPoR reduction approach can mitigate CPR, as an additional analyses.

5 Limited Effectiveness of MPoR Reduction

This section examines the effectiveness of MPoR reduction as a risk mitigation approach. As a result, the effect of this approach may be limited depending on the transaction type and the nature of the jump.

It is generally known that MPoR (δ^*), the period from default to liquidation, is not uniform, and varies depending on the legal form of the default and the circumstances leading to the default.

Many derivative transactions are conducted based on ISDA Master Agreement developed by International Swaps and Derivatives Association (ISDA) (2002). This agreement refers to some definitions including the event of default (EOD) by International Swaps and Derivatives Association (ISDA) (2006) and International Swaps and Derivatives Association (ISDA) (2021). They also define the procedures for the counterparty defaults.

Andersen et al. (2017) mention that financial institutions do not immediately initiate the procedures in order to maintain their reputation. As a result of such policies, MPoR may be extended, thereby unintentionally increasing CPR.

On the other hand, they also show that by reducing MPoR financial institutions can mitigate CPR. A concrete method to reduce MPoR is to establish an operational framework for prompt default procedures. From a legal perspective, they can reduce MPoR entering into an amendment agreement bilaterally to refine the definition of EOD to accelerate the triggering of default.

Here, we conduct numerical experiments of the CPR indicators corresponding to a smaller MPoR.

Since in No-Jump scenario in the previous section the regulatory requirements are satisfied, this section examines the other three scenarios.

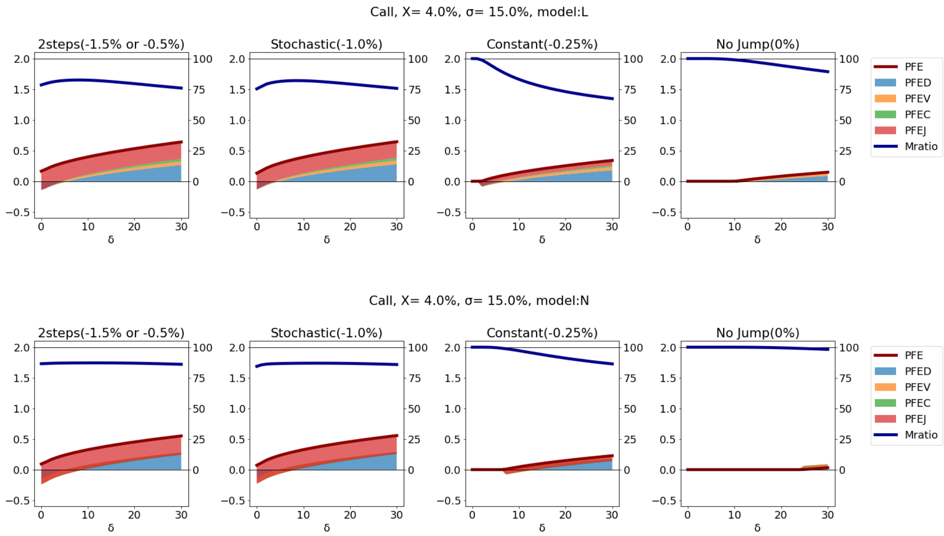

Fig. 8: By scenario of Jump, MPoR and CPR indicators. The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

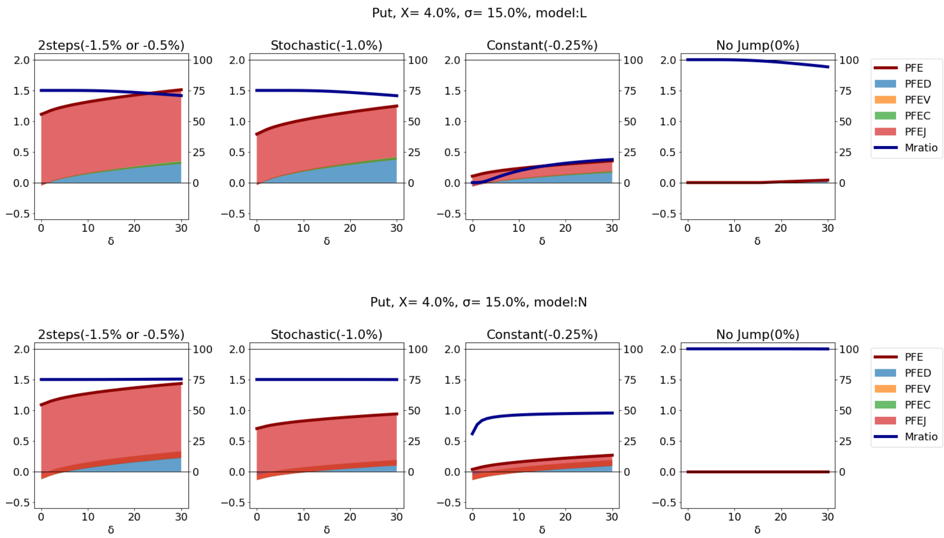

Fig. 9: By scenario of Jump, MPoR and CPR indicators .The scale of PFE is on the left axis, and that of Mratio is other (unit: %).

Fig. 8 and 9 show that PFE decreases due to the MPoR reduction. On the other hand, in Two-Steps and Stochastic scenarios MPoR reduction mitigates PFE of Call options but even when MPoR is reduced to zero, PFE does not necessarily decline to zero. In addition, it does not significantly mitigate PFE of Put options.

In the previous section we see that factor PFE of jump in these scenarios where the impact is probabilistic is larger. Since it cannot be eliminated by this approach, this approach cannot eliminate original PFE in these scenarios .

These analyses suggest that the effectiveness of the MPoR reduction approach may be limited depending on the transaction type and the nature of the jump and alternative approaches beyond MPoR reduction are required to mitigate CPR.

6 Conclusion

Traditionally, CPR has been discussed as an issue of risk management within individual financial institutions. Therefore, financial institutions negotiated to reflect their original risk management policies in the agreement on margin calculation and delivery. [14]

However, after the financial crisis (2008), the initiative in CPR management has shifted to the regulators, and uniform margin regulations have been implemented. Especially since the mandate of the regulations, IM calculation method is simplified and unified into ISDA SIMM.

On the other hand, it is generally known that a SIC default impacts financial markets. To incorporate the impact ISDA SIMM adopts the 1+3 standard as a conservative IM calculation method. But it does not accurately model the impact.

This study examines whether IM calculated by ISDA SIMM satisfies the regulatory requirements under the impact of the SIC default.

We model the impact as a jump in risk factors, derive approximate formulas for the CPR indicators, such as Mratio and PFE, and conduct numerical experiments under several jump scenarios.

The results show that Mratio often falls below 99% in the scenarios with jump, and PFE in the scenarios where the impact is probabilistic is larger.

It is verified that depending on the transaction type and the nature of the jump under the impact IM calculated by ISDA SIMM may not eliminate CPR to the regulatory level. Furthermore, it is crucial to accurately model the impact for the measurement and management of CPR.

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[14]Before the margin regulations, an independent amount(IA), which plays the same role as IM, is set. Although IA can be set even now, it cannot substitute for the IM regulations.

Appendix

A Derivation of the SABR model sensitivity

Two types of approximation formulas for a European option under the SABR model are proposed by Hagan et al. (2002), namely, the Log Normal type and Normal type. In this appendix, we derive the partial derivatives of each approximation formula.

A.1 Log Normal type

The approximation formula of Log Normal type is shown as follows:

$$
\begin{aligned}
V^{C(L)}(T-t, X(t), \sigma_L(t), K) &\simeq X(t)\Phi\left(d_+(t)\right) - K\Phi\left(d_-(t)\right), \\
V^{P(L)}(T-t, X(t), \sigma_L(t), K) &\simeq -X(t)\Phi\left(-d_+(t)\right) + K\Phi\left(-d_-(t)\right).
\end{aligned}
$$

where,

$$
\begin{aligned}
z_L(t) &:= z_L(X(t), \sigma_L(t), \beta, \nu, K) = \frac{(X(t)K)^{\frac{1-\beta}{2}} \ln \frac{X(t)}{K}}{\sigma(t)}\nu, \\
\mathfrak{X}(z, \rho) &:= \ln\left(\frac{\sqrt{1-2\rho z + z^2} + z - \rho}{1-\rho}\right), \\
\Theta_L(t) &:= \Theta_L(X(t), \sigma(t), \beta, \rho, \nu, K) = \frac{1}{24}\frac{\sigma^2(t)\left(1-\beta\right)^2}{\left(X(t)K\right)^{1-\beta}} + \frac{\rho\nu\sigma(t)\beta}{4\left(X(t)K\right)^{\frac{1-\beta}{2}}} + \frac{2-3\rho^2}{24}\nu^2,
\end{aligned}
$$

$$
\begin{aligned}
\sigma_L(t) &:= \sigma_L(T-t, X(t), \sigma(t), \beta, \rho, \nu, K) \\
&= \frac{\sigma(t)}{(X(t)K)^{\frac{1-\beta}{2}}\left[1 + \frac{(1-\beta)^2}{24}\left(\ln\frac{X(t)}{K}\right)^2 + \frac{(1-\beta)^4}{1920}\left(\ln\frac{X(t)}{K}\right)^4\right]} \frac{z_L(t)}{\mathfrak{X}(z_L(t), \rho)}\left[1 + \Theta_L(t)\left(T-t\right)\right],
\end{aligned}
$$

$$
d_+(t) := d_+(T-t, X(t), \sigma_L(t), K) = \frac{\ln\frac{X(t)}{K} + \frac{\sigma_L^2(t)(T-t)}{2}}{\sigma_L(t)\sqrt{T-t}}, \quad d_-(t) := d_+(t) - \sigma_L(t)\sqrt{T-t}.
$$

Before proceeding with determination of the partial derivatives, we first derive the relevant formulas for the standard normal distribution.

$$
\begin{aligned}
X(t)\phi\left(d_+(t)\right) = X(t)\frac{1}{\sqrt{2\pi}}e^{-\frac{d_+^2(t)}{2}} &= X(t)\frac{1}{\sqrt{2\pi}}e^{-\frac{d_-^2(t)}{2} - \ln\frac{X(t)}{K}} = K\phi\left(d_-(t)\right), \\
-\frac{d_+^2(t)}{2} = -\frac{1}{2}\left(\frac{\ln\frac{X(t)}{K}}{\sigma_L(t)\sqrt{T-t}} - \frac{\sigma_L(t)\sqrt{T-t}}{2}\right)^2 &- \ln\frac{X(t)}{K} = -\frac{d_-^2(t)}{2} - \ln\frac{X(t)}{K},
\end{aligned}
$$

$$\frac{\partial d_+}{\partial y}(t) = \frac{\partial}{\partial y}\left(\frac{\ln \frac{X(t)}{K}}{\sigma_L(t)\sqrt{T-t}} - \frac{\sigma_L(t)\sqrt{T-t}}{2}\right) + \sqrt{T-t}\sigma_{L,y}(t) = \frac{\partial d_-}{\partial y}(t) + \sqrt{T-t}\sigma_{L,y}(t).$$

Using these formulas, Delta (V_x), Vega (V_σ) and Theta (V_t) can be derived as follows,

$$\begin{aligned}
V_x^{C(L)}(t) &= \Phi\left(d_+(t)\right) + K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,x}(t),\\
V_x^{P(L)}(t) &= -\Phi\left(-d_+(t)\right) + K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,x}(t),\\
V_\sigma^{C(L)}(t) &= V_\sigma^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,\sigma}(t),\\
V_t^{C(L)}(t) &= V_t^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\sigma_{L,t}(t).
\end{aligned}$$

Also, Gamma (V_{xx}),Volga ($V_{x\sigma}$) and Vanna ($V_{\sigma\sigma}$) can be derived as follows.

$$V_{xx}^{C(L)}(t) = V_{xx}^{P(L)}(t) = \phi\left(d_+(t)\right)\frac{\partial d_+}{\partial X}(t) + K\phi\left(d_-(t)\right)\sqrt{T-t}\left(\sigma_{L,xx}(t) - d_-\frac{\partial d_-}{\partial X}(t)\sigma_{L,x}(t)\right),$$

$$\begin{aligned}
V_{x\sigma}^{C(L)}(t) &= V_{x\sigma}^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\left(\sigma_{L,x\sigma}(t) - d_-\frac{\partial d_-}{\partial X}(t)\sigma_{L,\sigma}(t)\right),\\
V_{\sigma\sigma}^{C(L)}(t) &= V_{\sigma\sigma}^{P(L)}(t) = K\phi\left(d_-(t)\right)\sqrt{T-t}\left(\sigma_{L,\sigma\sigma}(t) - d_-\frac{\partial d_-}{\partial \sigma}(t)\sigma_{L,\sigma}(t)\right).
\end{aligned}$$

where,

$$\begin{aligned}
\frac{\partial d_+}{\partial X}(t) &= \frac{1}{X(t)\sigma_L(t)\sqrt{T-t}} - \frac{d_-(t)}{\sigma_L(t)}\sigma_{L,x}(t), \quad & \frac{\partial d_+}{\partial \sigma}(t) &= -\frac{d_-(t)}{\sigma_L(t)}\sigma_{L,\sigma}(t),\\
\frac{\partial d_-}{\partial X}(t) &= \frac{1}{X(t)\sigma_L(t)\sqrt{T-t}} - \frac{d_+(t)}{\sigma_L(t)}\sigma_{L,x}(t), \quad & \frac{\partial d_-}{\partial \sigma}(t) &= -\frac{d_+(t)}{\sigma_L(t)}\sigma_{L,\sigma}(t).
\end{aligned}$$

Note that, the function σ_L and θ_L takes a singular point at $X = K$.

$$\begin{aligned}
\sigma_L(t) &= \frac{\sigma(t)}{X^{1-\beta}(t)}\left[1 + \theta_L(t)\left(T-t\right)\right],\\
\theta_L(t) &= \frac{1}{24}\frac{\sigma^2(t)\left(1-\beta\right)^2}{X^{2(1-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta}{4X^{1-\beta}(t)} + \frac{2-3\rho^2}{24}\nu^2.
\end{aligned}$$

In the case of $X = K$, the partial derivatives of the function σ_L and θ_L can be derived by:

$$\begin{aligned}
\sigma_{L,x}(t) &= \frac{\sigma(t)\left(\beta-1\right)}{X^{2-\beta}(t)}\left[1 + \theta_L\left(T-t\right)\right] + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,x}(t)\left(T-t\right),\\
\sigma_{L,\sigma}(t) &= \frac{1}{X^{1-\beta}(t)}\left[1 + \theta_L(t)\left(T-t\right)\right] + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,\sigma}\left(T-t\right),
\end{aligned}$$

$$
\begin{aligned}
\sigma_{L,t}(t) &= -\frac{\sigma(t)}{X^{1-\beta}(t)}\theta_L(t), \\
\sigma_{L,xx}(t) &= \frac{\sigma(t)\,(\beta-1)\,(\beta-2)}{X^{3-\beta}(t)}\left[1+\theta_L(t)\,(T-t)\right] \\
&\quad + 2\frac{\sigma(t)\,(\beta-1)}{X^{2-\beta}(t)}\theta_{L,x}(t)\,(T-t) + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,xx}(t)\,(T-t)\,, \\
\sigma_{L,x\sigma}(t) &= \frac{(\beta-1)}{X^{2-\beta}(t)}\left[1+\theta_L(t)\,(T-t)\right] + \frac{1}{X^{1-\beta}(t)}\theta_{L,x}(t)\,(T-t) \\
&\quad + \frac{\sigma(t)\,(\beta-1)}{X^{2-\beta}(t)}\theta_{L,\sigma}(t)\,(T-t) + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,x\sigma}(t)\,(T-t)\,, \\
\sigma_{L,\sigma\sigma}(t) &= 2\frac{1}{X^{\beta-1}(t)}\theta_{L,\sigma}(t)\,(T-t) + \frac{\sigma(t)}{X^{1-\beta}(t)}\theta_{L,\sigma\sigma}(t)\,(T-t)\,, \\
\theta_{L,x}(t) &= \frac{1}{12}\frac{\sigma^2(t)\,(\beta-1)^3}{X^{3-2\beta}(t)} + \frac{\rho\nu\sigma(t)\beta\,(\beta-1)}{4X^{2-\beta}(t)}, \\
\theta_{L,\sigma}(t) &= \frac{1}{12}\frac{\sigma(t)\,(\beta-1)^2}{X^{2-2\beta}(t)} + \frac{\rho\nu\beta}{4X^{1-\beta}(t)}, \\
\theta_{L,xx}(t) &= \frac{1}{12}\frac{\sigma^2(t)\,(\beta-1)^3\,(2\beta-3)}{X^{2(2-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta\,(\beta-1)\,(\beta-2)}{4X^{3-\beta}(t)}, \\
\theta_{L,x\sigma}(t) &= \frac{1}{6}\frac{\sigma(t)\,(\beta-1)^3}{X^{3-2\beta}(t)} + \frac{\rho\nu\beta\,(\beta-1)}{4X^{2-\beta}(t)}, \\
\theta_{L,\sigma\sigma}(t) &= \frac{1}{12}\frac{(\beta-1)^2}{X^{2(1-\beta)}(t)}.
\end{aligned}
$$

A.2 Normal type

The approximation formula of Normal type is also described as follows:

$$
\begin{aligned}
V^{C(N)}(T-t, X(t), \sigma_N(t), K) &\simeq (X(t)-K)\Phi(d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)), \\
V^{P(N)}(T-t, X(t), \sigma_N(t), K) &\simeq (K-X(t))\Phi(-d(t)) + \sigma_N(t)\sqrt{T-t}\phi(d(t)).
\end{aligned}
$$

where,

$$
\begin{aligned}
z_N(t) &:= z_N(X(t), \sigma(t), \beta, \nu, K) = \frac{\nu}{\sigma(t)}\frac{X(t)^{1-\beta}-K^{1-\beta}}{1-\beta}, \\
\mathfrak{X}(z,\rho) &:= \ln\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right), \\
\Theta_N(t) &:= \Theta_N(X(t), \sigma(t), \beta, \rho, \nu, K) \\
&= \frac{1}{24}\frac{\sigma^2(t)\beta(\beta-2)(1-\beta)^2\left(\ln\frac{X(t)}{K}\right)^2}{(X^{1-\beta}(t)-K^{1-\beta})^2} + \frac{\rho\nu\sigma(t)}{4}\frac{X^{\beta}(t)-K^{\beta}}{X(t)-K} + \frac{2-3\rho^2}{24}\nu^2,
\end{aligned}
$$

$$
\begin{aligned}
\sigma_N(t) &:= \sigma_N(T-t, X(t), \sigma(t), \beta, \rho, \nu, K) \\
&= \frac{\sigma(t)(1-\beta)(X(t)-K)}{X^{1-\beta}(t)-K^{1-\beta}} \frac{z_N(t)}{\mathfrak{X}(z_N(t),\rho)} \left[1+\Theta_N(t)(T-t)\right], \\
d(t) &:= d(T-t, X(t), \sigma_N(t), K) = \frac{X(t)-K}{\sigma_N(t)\sqrt{T-t}}.
\end{aligned}
$$

According to Kitani and Nakagawa (2024),

$$
\frac{\partial}{\partial y}\phi\left(d(t)\right) = -\frac{X(t)-K}{\sigma_N(t)\sqrt{T-t}}\phi\left(d(t)\right)\frac{\partial d}{\partial y}(t).
$$

Thus, we can derive Delta (V_x), Vega (V_σ) and Theta (V_t) as follows,

$$
\begin{aligned}
V_x^C(t) &= \Phi\left(d(t)\right) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,x}(t), \\
V_x^P(t) &= -\Phi\left(-d(t)\right) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,x}(t), \\
V_\sigma^C(t) &= V_\sigma^P(t) = \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,\sigma}(t), \\
V_t^C(t) &= V^P t(t) = \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,t}(t) - \frac{\sigma_N \phi\left(d(t)\right)}{2\sqrt{T-t}}.
\end{aligned}
$$

From Kitani and Nakagawa (2024),

$$
\begin{aligned}
&\frac{\partial}{\partial y}\phi(d) = -d\frac{\partial d}{\partial y}\phi(d), \\
&\frac{\partial d}{\partial X}(t) = \frac{1}{\sigma_N(t)\sqrt{T-t}}\left(1-\frac{X(t)-K}{\sigma_N(t)}\sigma_{N,x}(t)\right), \quad \frac{\partial d}{\partial \sigma}(t) = -\frac{X(t)-K}{\sigma_N^2(t)\sqrt{T-t}}\sigma_{N,\sigma}(t).
\end{aligned}
$$

Using the formula below.

$$
\frac{\partial}{\partial X}\left(-\Phi(-d)\right) = \frac{\partial}{\partial X}\Phi(d)
$$

In this case, Gamma (V_{xx}), Volga ($V_{x\sigma}$) and Vanna ($V_{\sigma\sigma}$) can be derived as follow,

$$
V_{xx}^C(t) = V_{xx}^P(t) = \left(\frac{X(t)-K}{\sigma_N(t)}\sigma_{N,x}(t)-1\right)^2 \frac{\phi\left(d(t)\right)}{\sigma_N(t)\sqrt{T-t}} + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,xx}(t),
$$

$$
\begin{aligned}
V_{x\sigma}^C(t) &= V_{x\sigma}^P(t) \\
&= \left(\frac{X(t)-K}{\sigma_N(t)}\sigma_{N,x}(t)-1\right)\phi\left(d(t)\right)\frac{X(t)-K}{\sigma_N^2(t)\sqrt{T-t}}\sigma_{N,\sigma}(t) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,x\sigma}(t),
\end{aligned}
$$

$$V^C_{\sigma\sigma}(t) = V^P_{\sigma\sigma}(t) = \frac{(X(t)-K)^2}{\sigma_N^3(t)\sqrt{T-t}}\phi\left(d(t)\right)\sigma^2_{N,\sigma}(t) + \sqrt{T-t}\phi\left(d(t)\right)\sigma_{N,\sigma\sigma}(t).$$

Same as the Log Normal type, the function σ_N and θ_N also take a singular point at $X = K$.

$$\begin{aligned}
\sigma_N(t) &= \sigma(t)X^{\beta}(t)\left[1+\theta_N(t)\left(T-t\right)\right],\\
\theta_N(t) &= \frac{1}{24}\frac{\sigma^2(t)\beta\left(\beta-2\right)}{X^{2(1-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta}{4X^{1-\beta}(t)} + \frac{2-3\rho^2}{24}\nu^2,
\end{aligned}$$

In the case of $X = K$, the partial derivatives of the function σ_L and θ_L can be derived by:

$$\begin{aligned}
\sigma_{N,x}(t) &= \sigma(t)\beta X^{\beta-1}(t)\left[1+\theta_N\left(T-t\right)\right] + \sigma(t)X^{\beta}(t)\theta_{N,x}(t)\left(T-t\right),\\
\sigma_{N,\sigma}(t) &= X^{\beta}(t)\left[1+\theta_N(t)\left(T-t\right)\right] + \sigma(t)X^{\beta}(t)\theta_{N,\sigma}\left(T-t\right),\\
\sigma_{N,t}(t) &= -\sigma(t)X^{\beta}(t)\theta_N(t),\\
\sigma_{N,xx}(t) &= \sigma(t)\beta\left(\beta-1\right)X^{\beta-1}(t)\left[1+\theta_L(t)\left(T-t\right)\right] + \sigma(t)\beta X^{\beta-1}(t)\theta_{L,x}(t)\left(T-t\right)\\
&\quad + \sigma(t)\beta X^{\beta-1}\theta_{L,x}(t) + \sigma(t)X^{\beta}(t)\theta_{L,xx}(t)\left(T-t\right),\\
\sigma_{N,x\sigma}(t) &= \beta X^{\beta-1}(t)\left[1+\theta_N(t)\left(T-t\right)\right] + X^{\beta}(t)\theta_{N,x}(t)\left(T-t\right)\\
\sigma_{N,\sigma\sigma}(t) &= 2X^{\beta}(t)\theta_{N,\sigma}(t)\left(T-t\right) + \sigma(t)X^{\beta}(t)\theta_{N,\sigma\sigma}(t),\\
&\quad + \sigma(t)\beta X^{\beta-1}(t)\theta_{N,\sigma}(t)\left(T-t\right) + \sigma(t)X^{\beta}(t)\theta_{N,x\sigma}(t)\left(T-t\right).
\end{aligned}$$

where,

$$\begin{aligned}
\theta_{N,x}(t) &= \frac{1}{12}\frac{\sigma^2(t)\left(\beta-1\right)\beta\left(\beta-2\right)}{X^{3-2\beta}(t)} + \frac{\rho\nu\sigma(t)\beta\left(\beta-1\right)}{4X^{2-\beta}(t)},\\
\theta_{N,\sigma}(t) &= \frac{1}{12}\frac{\sigma(t)\beta\left(\beta-2\right)}{X^{2(1-\beta)}(t)} + \frac{\rho\nu\beta}{4X^{1-\beta}(t)},\\
\theta_{N,xx}(t) &= \frac{1}{12}\frac{\sigma^2(t)\left(\beta-1\right)\beta\left(\beta-2\right)\left(2\beta-3\right)}{X^{2(2-\beta)}(t)} + \frac{\rho\nu\sigma(t)\beta\left(\beta-1\right)\left(\beta-2\right)}{4X^{3-\beta}(t)},\\
\theta_{N,x\sigma}(t) &= \frac{1}{6}\frac{\sigma(t)\beta\left(\beta-1\right)\left(\beta-2\right)}{X^{3-2\beta}(t)} + \frac{\rho\nu\beta\left(\beta-1\right)}{4X^{2-\beta}(t)},\\
\theta_{N,\sigma\sigma}(t) &= \frac{1}{12}\frac{\beta\left(\beta-2\right)}{X^{2(1-\beta)}(t)}.
\end{aligned}$$

Additionally, the function σ_N and z_N of the Normal type also take a singular point at $\beta = 1$.

$$\begin{aligned}
\sigma_N(t) &= \frac{\nu(X(t)-K)}{\mathfrak{X}(z_N,\rho)}\left[1+\theta_N(t)\left(T-t\right)\right],\\
z_N(t) &= \frac{\nu}{\sigma}\ln\frac{X(t)}{K}, \quad \theta_N(t) = -\frac{\sigma^2(t)}{24} + \frac{\rho\nu\sigma(t)}{4} + \frac{2-3\rho^2}{24}\nu^2.
\end{aligned}$$

B Conditional Expectation

For simplicity, we denote PE_D and PE_V in equation (8) by X_1 and X_2, respectively. We assume that X_1 and X_2 follow a normal distribution with means μ_1 and μ_2, variance σ_2^2 and σ_2^2. Each X_i can be expressed in trems of standard normal variables Z_i as:

$$Z_i = \mu_i + \sigma_i Z_i \quad i = 1, 2.$$

The corrlation coefficient between X_1 and X_2 (equivalently between Z_1 and Z_2) is denoted by ρ. Then, $\mathrm{PE}_Z = \mathrm{PE}_D + \mathrm{PE}_V$ can be expressed as,

$$Y = X_1 + X_2.$$

Based on this representation, we derive the expectation μ_Y and variance σ_Y^2 of Y [15] and the correlation coefficient $\rho_{1,Y}$ between X_1 and Y, as follows.

$$\begin{aligned}
&Y = \mu_Y + \sigma_Y Z_Y, \\
&\mu_Y = \mu_1 + \mu_2, \quad \sigma_Y^2 = \sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2, \\
&\rho_{1,Y} = \frac{\mathrm{Cov}\left(X_1, Y\right)}{\sigma_1\sigma_Y} = \frac{\mathrm{Cov}\left(X_1, X_1 + X_2\right)}{\sigma_1\sigma_Y} = \frac{\sigma_1^2 + \rho\sigma_1\sigma_2}{\sigma_1\sigma_Y}.
\end{aligned}$$

Then we calculate the following conditional expectation.

$$\begin{aligned}
\mathbf{E}\left[X_1 \,\middle|\, X_1 + X_2 = A\right] &= \mathbf{E}\left[\mu_1 + \sigma_1 Z_1 \,\middle|\, Y = A\right] \\
&= \mu_1 + \sigma_1 \mathbf{E}\left[Z_1 \,\middle|\, \mu_Y + \sigma_Y Z_Y = A\right],
\end{aligned}$$

where A is a constant which represents PFE_Z.

Here, using a random variable Z which is independent of Z_Y, Z_1 is expressed as,

$$Z_1 = \rho_Y Z_Y + \sqrt{1 - \rho_Y^2} Z.$$

Then,

$$\begin{aligned}
\mathbf{E}\left[Z_1 \,\middle|\, \mu_Y + \sigma_Y Z_Y = A\right] &= \mathbf{E}\left[\rho_{1,Y} Z_Y + \sqrt{1 - \rho_Y^2} Z \,\middle|\, Z_Y = \frac{A - \mu_Y}{\sigma_Y}\right] \\
&= \rho_{1,Y} \frac{1}{\sigma_Y}\left(A - \mu_Y\right) = \frac{\sigma_1^2 + \rho\sigma_1\sigma_2}{\sigma_1\sigma_Y^2}\left(A - \mu_Y\right).
\end{aligned}$$

Finally, the equation is calculated.

$$\mathbf{E}\left[X_1 \,\middle|\, X_1 + X_2 = A\right] = \mu_1 + \frac{\sigma_1^2 + 2\rho_Y\sigma_2}{\sigma_Y^2}\left(A - \mu_Y\right).$$

[15] Y follow a normal distribution from the reproducibility of normal distribution.

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